# A TEXT BOOK OF OPTICS

*( For Degree Students )*

## PART II

# PHYSICAL OPTICS


BY

Prof. KRISHNAPADA GHOSH, M. Sc.

*Head of the Physics Department*
*Ramakrishna Mission Residential College*
*Narendrapur, 24 Parganas, West Bengal*

*Formerly Head of the Physics Department*
*Scottish Church College, Calcutta-6*



ART UNION

CALCUTTA.

1944.

*Published by :*
ART UNION
80/15, Grey Street, Calcutta-6.

*Printed by :*
SRI ANIL KUMAR ROY
Art Union Printing Works Private Ltd.
80/15, Grey Street, Calcutta-6.

# PREFACE

The present volume is the thoroughly revised form of its earlier edition. The scope of the book is that of the graduateship examination of the Calcutta and the other Indian Universities covering the syllabus of the two-years and the three-years degree course. It includes the principles of Physical or Wave Optics and experimental illustrations of these principles. Since the book is intended for students who have passed the Higher Secondary Examination, the mathematical treatment is of very elementary nature.

The first chapter of the book gives the history of the earlier attempts to explain the elementary facts of Physical optics. The second chapter deals with simple periodic motions and their combinations. In the same chapter properties of waves explaining facts of Geometrical optics, their interference and diffraction as well as Huygen's principle have been profusely illustrated by photographs of behaviour of ripples on water surface under different circumstances. This chapter forms a very important background for understanding of the principles of Physical optics. The third chapter explains the facts of Geometrical optics in terms of wave characteristics of light. Interference of light and some of its technical applications are discussed in the fourth chapter. The fifth chapter deals with the phenomena of diffraction of light. The next three chapters deal with polarisation of light, interference of polarised light, and rotatory polarisation. The last chapter is concerned with spectroscopy and allied phenomena.

To collect materials for the book I have consulted all the well-known text books and treatises on the subject. To the Authors and Publishers of these books I offer my grateful thanks.

# CONTENTS

## CHAPTER I

General Introduction. The corpuscular and the wave theory of light.

## CHAPTER II

**Wave motion**

Simple harmonic motion. Terms and definitions. Energy of a particle executing S.H.M. The ripple tank. Progressive or phase waves, wave length, Mathematical expression for Progressive waves. Characteristies of Progressive waves, Stationary waves. Flux of energy. Appendix to Chapter II. Composition of simple harmonic motion.

## CHAPTER III

**Huygen's Principle and Geometrical optics**

Recapitulation. Huygen's principle. Refraction through prism. Colour and dispersion. Reflection and refraction of waves at plane and spherical boundaries. Refraction through a lens.

## CHAPTER IV

**Interference of Light**

Principle of superposition. Interference of waves from two coherent sources. Characteristics of interference fringes. Conditions for interference. Methods of producing coherent sources. Determination of wave-length of light by Fresnel's biprism method. Displacement of fringes by a thin plate. Change of phase on reflection. Interference of rays reflected from a film. Newton's rings. Rings by transmitted light. Uses of interference fringes. Stationary light waves. Lippmann's Colour photography.

## CHAPTER V

**Diffraction of Light**

Interference and diffraction, Huygen's principle modified by Fresnel. Calculation of intensity due to plane wave front. Rectilinear propagation of light. Theory of zone plates. Diffraction at a straight edge. Diffraction at a narrow obstacle. Diffraction through a narrow rectangular aperture. Diffraction at a circular aperture. The plane transmission grating. Secondary maxima and minima. Width of the primary spectrum. Dispersion produced by a grating. Chromatic resolving power of a grating. Measurement of wave-length of light with grating. Determination of resolving power of grating.

## CHAPTER VI

### Double Refraction and Polarisation of Light

The crystalline medium. Crystallographic axis. Law of rational indices. The Miller indices. Symmetry properties of crystals. Uniaxial and biaxial crystals. Double refraction on birefringence. Positive and negative crystals.

Huygen's theory of double refraction. Propagation of plane waves in uniaxial crystals. Refractive index of crystals. Polarisation of light by reflection. Brewster's law. Polarisation of light by double refraction. Methods of producing plane polarised light. Nicol's prism. Foucault's prism. Rochon's prism. Wollaston' prism. The Cornu polariscope. The polaroid.

## CHAPTER VII

### Interference of Polarised Light

Interference of plane polarised light. Phase difference between two polarised emergent rays from a crytal. The quarter-wave plate. The half-wave plate. Elliptically and circularly polarised light. Comparison of plane, circularly and elliptically polarised light. Analysis of circularly and elliptically polarised light.

## CHAPTER VIII

### Rotatory polarisation

Rotatory polarisation. Rotatory dispersion. Tint of passage. Polarimeters. Specific and molecular rotations. Saccharimetry.

## CHAPTER IX

### Spectroscopy

The complete spectrum. The electromagnetic theory. Fleming's method of verification of the nature of electro-magnetic waves. Kinds of spectra. Characteristics of line spectra. The spectral series. The Rydberg constant. Bohr's theory of line spectra. Broadening of spectral lines. The absorption spectrum. The solar spectrum. The Doppler's principle. Applications of Doppler's principle to astronomy. Fluorescence. Theory of fluorescent radiation. Scattering of light. Photography with infra-red rays. Rayleigh's equation for scattering. The Raman effect. Characteristics of Raman lines. Phosphorescence.

# A TEXT BOOK OF OPTICS

( FOR DEGREE STUDENTS )

## PART II

# PHYSICAL OPTICS

OR

# WAVE OPTICS

CHAPTER I

## GENERAL INTRODUCTION

**1. Fundamental Facts of Optics.** The aim of this part of the subject of Optics is to understand the nature of light and to develop suitable theories to explain the experimental facts. Accordingly, we open the subject with enumeration of the fundamental facts of light which any proposed theory has got to explain. These are summarised below :

(*a*) **Rectilinear propagation of light.** In a physically homogeneous medium light from a luminous object proceeds outwards in straight lines with a definite velocity.

(*b*) **Reflection and refraction of light.** At the boundary between two media which are filled with different kinds of matter, a portion of light is reflected and another portion is refracted, the deviation of the refracted ray depending on the colour of the light.

(*c*) **Total reflection of light.** Light passing from an optically denser to an optically rarer medium may be totally reflected at the interface between the two media, if the angle of incidence in the denser medium exceeds a certain critical value.

(*d*) **Interference of light.** Two beams of light, when superposed, under suitable conditions can destroy each other so as to

produce darkness. This fact will be illustrated by the following experiments :

**Experiment of Grimaldi.*** The phenomenon of interference was first observed by Grimaldi, though Hooke also laid claim to have made the same discovery simultaneously. Grimaldi arranged two small openings A and B (Fig. I-1) very close together in a shutter and allowed two diverging beams of sunlight to enter into a dark room.

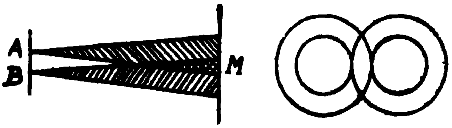



Fig. I–1

A screen was placed at such a distance from these holes that the two bright rings of light corresponding to the two diverging cones partially overlapped over the region *M*. He noticed the dark edge of one ring to be continuing round through the other, though over these portions the two beams of light were superposed.

But this appearance cannot be of physical origin, and it must be of physiological origin; so the appearance was an optical illusion.

**Young's experiment.** Fifteen years later, Thomas Young a member of medical profession modified Grimaldi's experiment and proved the interference phenomenon more decisively. He admitted a diverging beam of sunlight into a dark room through a small hole *S* in a shutter and received the beam on an opaque screen provided with two small holes A and B (Fig. I-2) very close together. On placing another screen at such a distance that the two beams partially overlapped, a series of alternately coloured and dark bands appeared in the overlapped region *M*. On stopping out one of the holes, the bands at once disappeared.

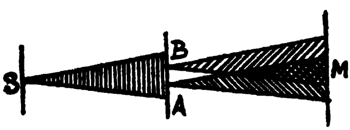



Fig. I–2

* Students are not supposed to be familiar with facts of Physical Optics. Hence it will be very helpful to the students if the experiments described here are shown to the class.

**Fresnel's experiment.** The next important advance in this direction was made by Fresnel, a young engineer of road construction. His active interest in Optics began in a solitary village, where he got a local locksmith to manufacture some imperfect apparatus for him. He had incomplete acquaintance with the optical literature of his times and it is only his genius and perseverance, and also friendly co-operation of Arago that soon raised him to the mastery over the subject and the Paris Academy soon honoured him by conferring upon him the Fellowship of the Academy.

Fresnel repeated Young's experiment, but replaced the two circular holes by two narrow vertical slits which effected a great improvement.

In all the experiments on interference referred to above the phenomenon of diffraction (to be presently described) was intermixed and Fresnel wanted to investigate the phenomenon of interference in a case which was free from the effects of diffraction. This led him to devise the famous mirror experiment which will be described in a later chapter.

**Colour of thin films. Hooke's observations.** Hooke observed that thin sections of mica showed brilliant colourations. He also noticed coloured fringes in the air interstices of the same mineral. He could produce these fringes by pressing together two plates of well-cleaned glass and proved that all transparent bodies, if obtained in sufficiently thin layers (such as thin blown glass, oil of turpentine, pitch, spirit of wine and bubbles etc.) showed similar colourations. He also proved that for showing such colouration the substance must have a limiting value of thickness; too thick or too thin layers do not show the effect. The most important point regarding his discoveries is that he was able to show that for production of such colours there must be an interaction of light reflected from the front and the back surfaces of the layer for, he showed that the polished surface of an opaque substance even though of proper thickness, did not show any colouration.

**Newton's rings.** In his 'Optick' of 1705 Newton described some experiments on the colour of this films. His simplest experi-

ment was to place on a double convex telescope objective a plano-convex lens with its plane side uppermost, so that the thickness of the enclosed layer of air increased from the centre outwards. He noticed a series of alternately dark and coloured concentric rings round the point of contact of the two lenses which appeared black when viewed by reflected light. On pressing the lenses, the whole system of bands moved outwards and, on releasing the pressure, they moved back and vanished at the centre. By means of a compass he could measure the diameter of the rings and showed experimentally that radius of a ring was inversely proportional to the thickness of the layer of air at the point where the ring was situated. Using monochromatic light he obtained a much larger number of rings. He also observed that the colour of the rings seen by transmitted light was complementary to that of the rings seen by reflected light.

Young filled the space between the two lenses by a liquid and by observing Newton's rings in the liquid he concluded that light has smaller velocity in a mediun in which it bends towards the normal after refraction.

(*e*) **Diffraction of light.** On meeting an obstacle, rays of light bend round the edge of the obstacle, as it were, so as to produce some illumination inside the geometrical shadow. The phenomenon of diffraction, therefore, stands in direct contradiction to rectilinear propagation of light. Hence any theory of light must try to correlate these two phenomena, or it should try to explain one of these phenomena in terms of the other which can be proved to be more fundamental than the other.

**Grimaldi's experiment.** It was observed by Grimaldi that the shadow of an obstacle placed in the path of sunlight admitted into a dark room through an aperture in a blind was much broader than what it would have been if the light were propagated in straight lines through the edges of the obstacle. He observed three coloured bands running parallel to the edge of the obstacle outside the geometrical shadow. With a strong source of illumination he also observed fringes within the geometrical shadow.

**Hooke's experiments.** Hooke showed some experiments to the Royal Society which were essentially an incomplete and inaccurate repetition of Grimaldi's experiment, though Hooke considered them to be origina .

**Newton's experiments.** Newton's experiments were more refined than Grimaldi's and he devised a greater variety of experiments. But, strange to say, he completely ignored the fringes within the geometrical shadow. He showed that the substance of the medium was not essential for the diffraction phenomena, for, he obtained diffraction fringes when a hair was interposed between two plates of glass. These fringes could also be produced by scratches in glass and also by transparent screens.

**Fresnel's experiments.** Fresnel proved that the phenomenon of diffraction does not depend on the degree of polish or sharpness of the edge of the obstacle; for he found that the fringes of the same character were obtained whether light passed over the sharp or the blunt edge of a razor. From this he concluded that diffraction was not due to interference of direct light with light reflected from the edge of the obstacle, as was supposed by Young.

(*f*) **Double refraction and polarisation of light. Experiment of Bartholinus.** If a crystal of ice-land spar or calcite be placed on a sheet of white paper marked with a dot and viewed directly from above, the dot appears to be doubled, one of which is undisplaced and the other displaced in a particular direction by an amount proportional to the thickness of the crystal. On rotating the crystal, keeping it at the same time resting on the paper, one of the images remained stationary, while the other appeared to rotate round the first. This was discovered by Bartholinus, who at once recognised an extra-ordinary type of refraction in the crystal, since, he found that a glass plate of similar shape and thickness would not behave in a similar way.

**Huygen's experiment.** Huygen passed a narrow beam of sunlight normally through the crystal. On emergence from the crystal, the beam was found to be separated into two parts.

Both of these beams emerged parallel to the direction of the incident ray. One of them was undisplaced and was called the **ordinary-ray** and the other suffered displacement by a certain amount and was called the **extra-ordinary ray.** On placing an exactly similar crystal with similar orientation as the first in the path of the beam, the ordinary beam emerged as ordinary and the extra-ordinary as extra-ordinary through the second crystal. But the lateral separation between the emergent rays was double. The same effect was observed, if the second crystal were rotated, through 180°. But when the second crystal was rotated only through 90°, the ordinary ray emerged as extra-ordinary ray and the extra-ordinary as ordinary ray through the second crystal. When the second crystal was placed at any other angle with respect to the first, four rays emerged through the second crystal, of which two were ordinary and the other two were extra-ordinary.

**Newton's observations.** Newton personally did not make any exhaustive experimental investigation on double refraction. From his observations Newton ascribes asymmetrical nature to rays and concludes that the ordinary and the extra-ordinary rays would differ from one another with respect to position only, *i.e.* each ray possesses a certain property on two parallel sides and different property on two mutually perpendicular sides; in other words, each ray after transmission through the crystal was **polarised**, the ordinary ray in one plane the extra-ordinary ray in the perpendicular plane.

**Experiment of Malus.** While engaged, in the problem of double refraction at his house in the Rue denfer, Malus chanced to observe the image of the setting sun reflected from the window of the Luxembourg palace through a calcite crystal and to his surprise he found that for certain orientations of the crystal only one image instead of two appeared, the ordinary and the extra-ordinary image disappearing in turn as the crystal was rotated. He then examined light from a candle reflected from a water surface and then from a glass surface and this twice reflected beam was observed through a calcite crystal. He also allowed light first to be transmitted through calcite and then reflected. He thus discovered all the essential features

of polarised light. Towards the end of 1809, **Malus** showed that not only reflected but also refracted light showed traces of polarisation. The most important of Malus' laws is the discovery of the cosine square law which we shall describe in detail in a later chapter.

**Brewster's experiments.** Brewster could produce a single polarised beam by transmission of light through a thick agate crystal cut perpendicular to its cleavage plane.

**Biot and Seebeck** discovered almost simultaneously with Brewster that a plate of tourmaline also shows double refraction, but one of the polarised components is absorbed by transmission.

**Fresnel and Arago,** then studied the conditions of interference of polarised light. Arago noticed the rotation of the plane of polarisation by quarz crystal and Biot discovered that the rotation may be left-handed or right-handed. In 1815 Biot discovered rotation of the plane of polarisation in liquids and he also rendered great service in the development of the knowledge of chromatic polarisation.

## 2. The Corpuscular Theory and the Wave Theory of Light.

**Light is immaterial.** The fact that two beams of light when superposed can, under suitable conditions, destroy each other is proved by the experiments on interference of light. This at once suggests that light cannot be of material nature, for two pieces of matter when placed together cannot destroy each other.

**Light is energy.** From the methods of production of light it appears that light is energy, for, to produce light some form of energy must disappear.

**Light moves with a finite velocity.** Experiments of Romer and also of other observers prove that light moves with a finite velocity.

The conclusion is that *light is energy and that it moves with a finite velocity.* Hence the question of propagation of light is connected with the question of transference of energy from one point in space to another.

**Two possible modes of propagation of energy.** We shall illustrate these methods by an example. Suppose a man on the shore wants to impart motion to a boat on a lake. To make the boat move be might fire bullets into it from the shore. Each bullet carrying some energy with it imparts motion to the boat on striking it. There is yet another method: he can excite waves in water; these waves after breaking on the boat would cause it to move up and down.

**2. The Corpuscular or the Emission Theory of Light.** The first method of propagation of energy as explained above corresponds to the corpuscular theory of light instituted by Newton. According to this theory a luminous body shoots out infinitely small particles or corpuscles with finite, though large, velocity. By their impact on the retina they excite the sensation of vision. These particles are supposed to be small masses of matter and, therefore, the paths of these particles are influenced by interaction with material media placed in the path of the particles. A luminous body generally emits different sized particles simultaneously. The shape of the particles is not exactly spherical but shows some amount of asymmetry.

We shall now proceed to examine how far the corpuscular theory is able to explain the fundamental facts of optics.

**Explanation of rectilinear propagation.** This follows as a necessary consequence of Newton's first law of motion, which states that a material particle moves uniformly in a straight line when it is not acted upon by any external force.

**Explanation of reflection.** This can be explained with the help of Fig. I-3 where $AB$ is the reflecting surface. When a corpuscle moving along $PQ$ comes within a certain distance $MN$ from the reflecting surface, it *begins to experience a force of repulsion in a direction normal to the surface and directed outwards.* As a result the component of its velocity parallel to the surface remains unaffected, while the component at $Q$ perpendicular to the surface gradually diminishes, becomes zero at the lowest point $R$ of its path, and then changes

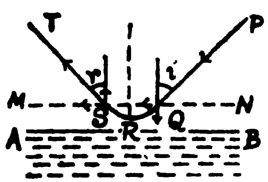


Fig. I-3

sign and is completely reversed at $S$, so that the velocity of the particle at $S$ is the same as that at $Q$. Let $v$ be the velocity the particle at $Q$ and $i$ the angle of incidence at the same point Let also $r$ be the angle of reflection at $S$. Hence resolving the velocity $v$ at $Q$ and $S$ parallel to $AB$ and equating them, we get

$$v \sin i = v \sin r$$

so that
$$i = r \qquad \ldots (2{\cdot}1)$$

or, the angle of incidence is equal to the angle of reflection.

**Explanation of refraction.** In this case, according to Newton, the corpuscle when it comes within a certain distance $MN$ from the surface experiences *a force of attraction in a direction normal to the surface and directed inwards*. This attraction continues up to a certain depth RS (Fig. I-4) inside the medium. Accordingly, the component of its velocity parallel to the surface remains unaltered while the component perpendicular to the surface $AB$ continuously increases from $Q$ to $S$.

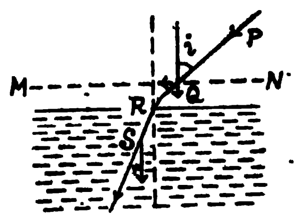


Fig. I-4

Hence the velocity of the corpuscle inside and outside the refracting medium are different. Let $v$ and $v'$ be the velocities of the corpuscle inside and outside the medium and let $i$ and $r$ be the angles of incidence and refraction respectively. Hence, resolving the velocities $v$ and $v'$ at $Q$ and $S$ parallel to the surface and equating them, we get

$$v \sin i = v' \sin r$$

or
$$\frac{\sin i}{\sin r} = \frac{v'}{v} = \text{constant} \qquad \ldots (2{\cdot}2)$$

in other words, the sine of the angle of incidence bears a constant ratio to the sine of the angle of refraction, which is Snell's law.

**Explanation of critical reflection.** To explain the phenomenon of critical reflection Newton assumed that a ray moving in an optically denser medium is subjected to a force of attraction normally inwards, so that it is only the normal component of the velocity of the corpuscle that gradually diminishes in magnitude. If $i$ be the angle of incidence and $v$ the velocity of the corpuscle at $Q$, (Fig. I-5) the component at $Q$ perpendicular to the surface is $v \cos i$. Since cosine of an angle diminishes as the angle increases, it follows that for a particular value of the angle of incidence, the component at $Q$ is of such a magnitude that it is completely balanced by the effect of the inward force when the corpuscle arrives at $R$. Under this circumstance the corpuscle cannot emerge out of the surface and it is totally reflected into the medium.

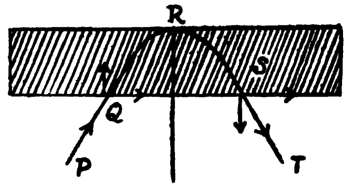


Fig. I-5

**Explanation of dispersion.** To explain the phenomenon of dispersion, we start with Newton's observation "lights which differ in clour differ also in degree of refrangibility." The difference in refrangibility, according to the corpuscular theory is caused by the difference in mass and size of the corpuscles. Accordingly, a beam of white light would be composed of an assemblage of differently sized corpuscles. When such a beam of light is refracted into a medium, the particles of different mass are attracted differently, and, therefore, they are bent towards the normal by different amounts.

**Difficulties of the corpuscular theory** (*a*) **Velocity of light in denser medium.** From Equation (2·2) it follows that if $i$ is greater than $r$, so that sin $i$ is also greater than sin $r$ then $v'$ is greater than $v$. In other words, the velocity of light in a medium in which the refracted ray bends towards the normal is greater than in a medium in which the bending is less. *i.e.* velocity of light in an optically denser medium is greater than in an optically rarer medium. This is contrary to all experimental results.

(*b*) If the corpuscles are of material nature, the body emitting light would lose its weight.

(*c*) The velocity of the corpuscles would depend on the temperature of the source in accordance with the kinetic theory.

(*d*) When a beam of light is incident on a refracting surface, light is simultaneously reflected and refracted. We have already seen that reflection is effected by the force of repulsion between the refracting body and the corpuscles while for refraction the same body must exert a force of attraction on the corpuscles. Why some of the corpuscles are attracted and others are repelled by the same surface ? To explain this anomalous behaviour of the corpuscles Newton had to attribute to light "*periodic fits of easy transmission and reflection.*"

**Newton's theory of 'fits'.** According to Newton periodic fits were associated with the light as soon as it left the luminous body and these fits followed at regular intervals. The simultaneous reflection and refraction was explained by Newton in the following way:

"As Stones by falling upon Water put the Water into an undulatory Motion, and all bodies by percussion excite vibration in the air, so the Rays of light by impinging on any refracting or reflecting Surface excite vibrations in refracting or reflecting Medium or Substance much after the manner that vibrations are propagated in the Air for causing sound and move faster than the Rays so as to overtake them, and that when any ray which is in that part of the vibration which conspires with Motion, it easily breaks through a refracting Surface, but when it is in the contrary part of the vibration which impedes its Motion, it is easily reflected......But whether the hypothesis is true or false I do not here consider."

It is interesting to note that Newton here recognised some kind of periodicity of light.

**Explanation of interference of light. Colour of thin films.** This was explained by Newton with the help of his theory of fits. The fraction of the incident light which is reflected

at the first surface of a lamina may, following Newton, be neglected. If a fit of transmitted fraction of a ray arrives at the second face of the lamina at the same time as the next following fit arrives at the first, that is, after a complete period, then the former is transmitted by the second face and the lamina consequently remains dark. If, however, the light arrives at the second face after a half period has elapsed, it is reflected so that the lamina appears bright.

Thus, for explanation of some optical phenomena the corpuscular theory alone was not sufficient; Newton had to make other subsidiary assumptions which are again incomprehensible.

**Explanation of diffraction.** Newton could not give a definite explanation of the phenomenon of diffraction, neither he recognised the phenomenon of diffraction, for he explicitly stated that in Grimaldi's experiment, as represented by him, the fringes existed only outside the geometrical shadow. Nevertheless, he concludes his Optics with few interrogations such as: Do not bodies act upon light at a distance ? Do not rays which differ in refrangibility differ also in flexibility ?

**Explanation of double refraction.** To explain this Newton assumed that the four sides of ray were probably endowed with forces which exhibited polarity like magnetic forces, for example, which could interact with matter in a large variety of ways according to the differences in the latter.

Thus the corpuscular theory is rather vague and is not self-sufficient. It has, therefore, been abandoned more particularly because it sometimes leads to results which are contrary to experiences.

**3. The Wave Theory of Light.** The corpuscular theory of light as explained above being inadequate to explain all the observed facts, the only possible alternative theory must be the theory of transmission of energy by means of waves. According to the wave theory of light a luminous body excites some kinds of waves in an all-pervading medium called the **ether**. These waves are propagated in all directions and on falling on the retina excites the sensation of vision.

It appears from his writings that the non-uniform and periodic distribution of light round the edges of an obstacle probably reminded Grimaldi of similar occurrence when water flows round an obstacle projecting out of its surface.

In discussing the properties of light Hooke also ascribed wave character to light; for, he says—"fiery (luminous) bodies are engaged in violent motion between their parts. This light-producing motion cannot be translational, for, all the parts of the body remain together, neither, it can be rotational motion, for this is characteristic of liquids; it must, therefore, be vibratory motion of very short period."

Huygens, Euler, and Hartley instituted the wave hypothesis of light. We shall devote some space here to discuss Huygens' assumptions, these being of special interest in the later stages of development of physical optics.

**Wave theory of Huygens.** According to Huygens, individual points of a luminous body excites waves in ether which readily permeates all matter. This medium, according to him, consists of elastic spheres which can impart their impulse from one to the other. He illustrated his ideas with the help of a row of contiguous spheres. When a velocity was imparted to the first sphere, it was transmitted to the last in a short but finite time, all the intermediate spheres remaining at rest. Such impulses could traverse the spheres in opposite directions simultaneously and cross one another. He imagined light waves were longitudinal like perfectly symmetrical sound waves.

**Huygens' waves.** We know from experience that a circular wave generated by dropping a stone on water surface grows into a circle of ever-increasing radius. A second wave then follows, then a third, and in a short time a series of waves are generated following each other which spread out uniformly over the surface of water. The idea of such a regular sequence of equidistant waves spreading outwards from a source of disturbance was alien to Huygens. According to Huygens, the irregularly pulsating point of luminous body produces impulses which are imparted not only in one direction but to all particles of ether

in contact with the pulsating one, so that the spreading out of the impulse occurs simultaneously on all sides in the shape of spheres.

Their effects become noticeable only when several weak pulses coming from different centres of the luminous source add up or unite to form a stronger wave, which is the envelope of the weaker waves at a given instant.

His idea of propagation of waves probably originated from the observation of wedge-shaped waves produced by a stick moving rapidly through water surface in one direction or from the bow of a moving boat. This idea can be illustrated with the help of Fig. I-6. In this figure $AM$ shows the line of motion of the stick. As the stick moves through the surface of water, the particles $A$, $B$, $C$, $D$ etc., lying on the line of motion of the stick

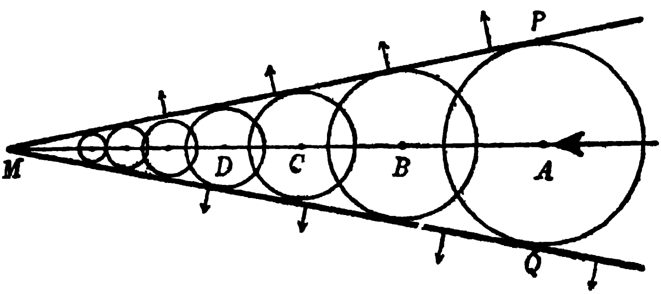



Fig. I-6

are successively disturbed and circular waves start from each of these disturbed points in succession. Let $V$ be the velocity of propagation of the waves in water and tle $v$ be the velocity of the stick. Let $t$ be the time taken by the stick to move from any point $A$ to $M$. Then $AM = v.t.$ The circular wave generated at $A$ grows into a circle of radius $AP = V.t.$ after time interval $t$ during which the stick moves from $A$ to $M$. Hence

$$\frac{AP}{AM} = \frac{V}{v} \ (=\text{constant}) \ \text{ or, } AP = \frac{V}{v} \ . \ AM \qquad \ldots(3.1)$$

Since $V/v$ is constant for all points on AM the relation (3.1) is true for every other point. Hence, the radius of each indivi-

instant *t* is proportional to its distance from *M*, so that all the spheres touch over two common tangents *MP* and *MQ* which is, therefore, the grand wave. When the stick moves through the surface of water, the individual weak waves sent out by the points *A*, *B*, *C*, *D*, etc., are scarcely visible, it is the wedge-shaped envelope *PM* which is found to be spreading outwards from the stick.

**Huygens' method of explaining rectilinear propagation.** The most important point in Huygens' conception is the novel idea of mechanism of spreading of waves from one region to another. According to him, *each point of the grand wave at any instant is the centre of secondary spherical waves. The envelope of all these secondary waves at a later instant is the new wave front at that instant.*

This is *Huygens' principle.* This means that Huygens draws no distinction between the first particle which receives and transmits the impulse and the particle to which the impulse has been communicated through the agency of a series of other particles. This is the most important principle in Physical Optics, since it provides an easy method of mathematical handling of an optical problem.

**Rectilinear propagation of light.** With the help of this principle Huygens proceeds to prove that propagation of light must be necessarily rectilinear. This can be easily done with the help of Fig. I-7. In this figure, O represents a point source of light. When the radius of the grand wave has reached a certain value, let a part of the wave be cut off by a screen provided with a circular hole at *AB*. According to Huygens' principle, each of the points *a*, *b*, *c*, *d* etc., of the grand wave at *AB* are sources of secondary waves. Round each of these points he draws spherical waves of the same radius. It is evident that *the*

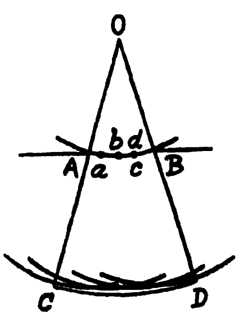


Fig. I-7

Outside the cone, the waves will reach by scattering and will, therefore, be ineffective.

**Huygen's method of Construction for the reflected and refracted waves.** This is given in Fig. I-8. In this figure *AM* represents the trace of the surface of separation between two media *a* and *b* by the plane of the paper. *MP* is the trace of the incident wave which is supposed to be

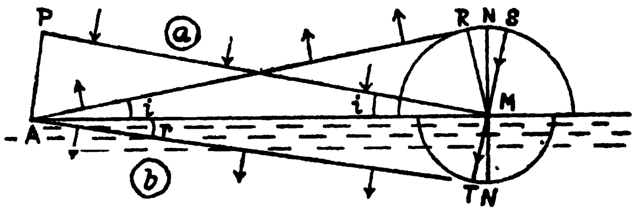



Fig. I-8

plane. The incident wave is moving parallel to itself in the direction *PA*. As the wave front *PM* moves parallel to itself, the point of intersection *M* of *PM* with *AM* moves in the direction *M* to *A*. These points of intersection become successive centres of Hughen's secondary wave. Let $v$ be the velocity of propagation of wave in the medium *a* and $v'$ its velocity in the medium *b*. Let $t$ seconds be the time taken by the incident wave to move from *P* to *A*. It is evident that the point *A* will be disturbed $t$ seconds after the point *M* has been disturbed. Hence, the radius of the secondary wave at *M* in the medium *a* will acquire a value $v.t.$ ($= AP$) at the instant when the particle at *A* is just on the point of being disturbed. Similarly, the secondary wave at *M* in the medium *b* will acquire a value $v't$ at the same instant. With *M* as centre and radii equal to $v.\ t$ and $v.'t$ draw two hemispheres in the media *a* and *b* respectively. Draw two tangent planes *AR* and *AT* to touch these two spheres. Then *AR* and *AT* are respectively the reflected and refracted waves. [See also Fig. 4, Plate I and Fig. 5, Plate II.]

**Proof of Snell's law.** Let $i$ and $r$ be the angles of inclination of the incident and refracted waves to the surface of

separation $AM$. Then, since the triangles $ARM$ and $ATM$ are identical by construction,

$$\sin i=\frac{RM}{AM}=\frac{AP}{AM}=\frac{v.t.}{AM}$$

and $$\sin r=\frac{MT}{AM}=\frac{v.'t}{AM}$$

$\therefore$ $$\frac{\sin i}{\sin r}=\frac{v}{v'} \qquad \ldots(3{\cdot}2)$$

Now, *a ray according to the wave theory, is the direction of propagation of a wave.* In an isotropic medium a ray is always normal to the wave front. Hence the line SM which is perpendicular to the incident wave PM must be an incident ray at M. For the same reason $MR$ and $MT$ must be the corresponding reflected and refracted rays respectively at M. Let NMN$'$ be the normal to the refracting surface at M. Then, it is evident from the geometry of the figure that $\angle$NMS$=i$, $\angle$N$'$MT$=r$. Hence, $i$ and $r$ are respectively the angles of incidence and refraction. Thus, we have, for the refractive index,

$$\mu=\frac{\sin i}{\sin r}=\frac{v}{v'},=\text{constant} \qquad \ldots(3{\cdot}3)$$

which is Snell's law.

**Proof of the law of reflection.** From the identical triangles ARM and APM (Fig. I-8), $\angle$AMP $=\angle$RAM. Also $\angle$NMR$=\angle$RAM. If follows that $\angle$RMN$=\angle$SMN or, the angle of incidence is equal to the angle of reflection.

**Explanation of critical reflection.** In Fig. I-9, velocity of waves in the medium '*b*' is less than that in the medium '*a*'. Let us now reverse the case and let us suppose that the incident wave lies in the denser medium *b*. Then the construction for the reflected and refracted waves will be as shown in Fig. I-9.

In this figure MP is the incident wave, AR and AT are the corresponding reflected and refracted waves. Since the velocity $v'$ in medium '*b*' is less than that in the medium '*a*', MR ($=v't$)

is always less than MT ($=vt$). As the inclination of the incident wave increases, the radius of the lower circle also increases and therewith the radius of the upper circle increases still further.

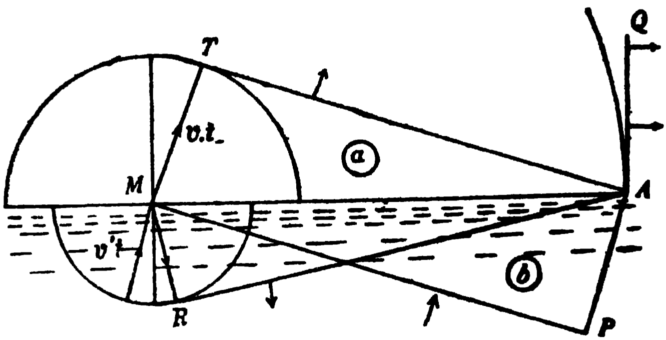



Fig. 1-9

It may happen that for a particular angle of incidence, the radius of the upper circle is greater than AM. In that case the tangent AT cannot be drawn and there can be no refracted wave in the upper medium, all the light being totally reflected.

The limiting condition is reached when the radius of the upper circle is equal to AM. In this case the refracted wave-front is perpendicular to the refracting surface at A and the refracted ray is parallel to the refracting surface so that the angle of refraction is 90°. Hence, if $\theta$ and $r$ are the angles of incidence and refraction under the limiting condition

$$\frac{1}{\mu}=\frac{\sin\theta}{\sin r}=\frac{\sin\theta}{\sin 90^\circ}=\frac{v'}{v} \qquad \ldots(3\cdot4)$$

or, $$\sin\theta=\frac{1}{\mu}.$$

The principal fruits of Huygens' labour lay in the explanation of rectilinear propagation, reflection, simple and double refraction from the rate of propagation of light. He never tried to explain the phenomena of interference and diffraction. Huygens' method of explanation of double refraction will be discussed in detail in a later chapter.

**Merits and demerits of Huygen's method.** (*a*) Comparing Equations (2·1) and (2·3) we see that while corpuscular theory of light required the velocity of light in an optically denser medium to be greater than in a rarer medium, Huygens' wave theory required a contrary result, which is in accord with our experience. Hence we conclude that Huygen's line of attack of the problem was in the right direction. But

(*b*) Huygens necessarily disregarded the effects of diffraction or supposed them to be absent.

(*c*) The intensity of each ray as it passes on its way must be continually weaker owing to the loss of impulse laterally.

(*d*) If each point of a wave be the centre of another wave, the latter would necessarily radiate waves in all directions, that is, even in a homogenous medium light waves would move in the backward as well as in the forward direction which is contrary to experiences.

(*e*) Since Huygens' waves are longitudinal, Huygens' theory offers no explanation of the phenomenon of polarisation.

**Young's contribution.** In the meantime while some philosophers were busy with speculations regarding the nature of light another branch of Physics, namely Physical Acoustics was being rapidly developed. In Huygens-Newton's time all the necessary elements of facts of Acoustics and Optics were known and they only required correlation.

Hundred years after Huygens and Newton had established their theories, Thomas Young entered the field and began the study of Optics and Acoustics. He recognised numerous instances of similarity between the behaviours of light and sound. As a result of his investigations he was led to the conclusion that all the optical phenomena could be explained on the wave hypothesis and he cautiously laid down the principle of interference as follows :

"*As in the case of sound (a musical note) so also in a (coloured) beam of light there follows a succession at regular intervals of alternately opposite properties (states) which are mutually able to destroy (neutralise) each other.*"

Unfortunately a scientific reactionary who felt himself called upon to uphold Newton's theory commenced an unparalleled attack upon Young and the Royal Society. This marred his reputation for many years and it was recovered by the labours of Fresnel later on. In recognition of the services rendered to the cause of science the Rumford Medal was consigned to the dying Fresnel in the year 1827 and the same year Young was elected foreign member of the National Institute of France.

**Fresnel's contribution.** Fresnel combined Huygens' theory of spreading of waves in *all* directions with Young's principle of interference as laid down above. By such combination he was able to modify Huygen's theory. This *modified Huygens' theory*, as it is called, is the starting point of explanation of all optical phenomena namely interference, diffraction and polarisation and this will form the sujbect matter of this book.

Before we take up the subject proper, it will be useful to make the reader familiar with the properties of waves. Since all waves behave in the same way, we shall study the properties of waves with the help of water waves particularly ripples. Again, periodicity and wave motion being inseparably connected together we shall begin with the study of periodic motion. This will form the subject-matter of the next chapter.

# CHAPTER II

## WAVE MOTION

Since in this book we shall be chiefly concerned with interactions of simple harmonic waves, we open this chapter with the characteristics of simple harmonic motion.

**4. Definition of simple harmonic motion.** A body is said to execute harmonic motion, if its time-displacement curve is a harmonic curve, that is a sine curve or a cosine curve.

Accordingly, if $y$ is the displacement of a particle executing simple harmonic motion from its rest position at an instant $t$, then its equation of motion is

$$y = a \sin (\omega t - \delta) \qquad \ldots(4{\cdot}1)$$

where $a$, $\omega$ and $\delta$ are constants

**Velocity of a particle executing simple harmonic motion.** Let $y$ be the displacement of a particle executing S.H.M. at instant $t$. Then its velocity at the instant $t$ is

$$v = \frac{dy}{dt} = a\omega\cos(\omega t - \delta) \qquad \ldots(4.2)$$

**Acceleration of a particle executing simple harmonic motion.** Let $v$=the velocity of the above particle at the instant $t$.

Now, acceleration is the rate of change of velocity with respect to time, so that, the acceleration of the particle at the instant $t$ is

$$f = \frac{dv}{dt} = -a\omega^2 . \sin (\omega t - \delta)$$

or, since $y = a \sin (\bar{\omega} t - \delta)$, $f = -\omega^2 y$. $\qquad (4{\cdot}3)$

The negative sign in Eqn. (4·3) indicates that the acceleration is always opposed to the direction of displacement. Hence, it is always directed towards the centre or the position of equilibrium.

**Characteristics of simple harmonic motion.** Eqn. (4·3) gives the two most important characteristics of simple harmonic motion :

(1) *the acceleration of a particle executing simple harmonic motion is always proportional to its displacement from the mean position* ;

(2) *the acceleration of a particle executing simple harmonic motion is always directed towards its mean position.*

If $m$ be the mass of the particle executing simple harmonic motion, the force acting on the particle is

$$P = -m\omega^2 y \qquad \ldots \quad (4\cdot4)$$

The force is, therefore, proportional to the displacement and is always directed towards the centre. This force which tends to restore equilibrium is called the **force of restitution** or the **controlling force.** *A particle executes simple harmonic motion when there is a force of restitution proportional to its displacement.*

We have obtained three very important relations obeyed by a particle executing simple harmonic motion. These are :

the displacement at instant $t$, $y = a \sin(\omega t - \delta)$

,, velocity ,, ,, ,, $v = a\omega \cos(\omega t - \delta)$

,, acceleration ,, ,, ,, $f = -a\omega^2 \sin(\omega t - \delta)$

**5. Terms and Definitions. Amplitude of vibration.** The maximum displacement of a vibrating particle on either side of its mean position is called its amplitude of vibration. It follows from Eqn. (4·1) that $a$ is the amplitude of vibration.

**Complete oscillation.** A vibrating particle is said to execute a complete oscillation when it passes through any point on its path in a given direction and back through the same point in the same direction for the next time.

**Period of oscillation.** Time taken by a vibrating particle to execute one oscillation is called the period of oscillation.

In other words, the displacement of the particle executing simple harmonic motion is the same after each complete time period . Let $T$ be the time-period of oscillation. Then the displacement of the particle at instants $t$ and $t+T$ would be the same. Let $y$ be the displacement of the particle at instant $t$. Then

$$y = a \sin (\omega t - \delta) \qquad \dots \quad (5\cdot1)$$

From what has been said above

or
$$y = a \sin \{\omega(t+T) - \delta\} \qquad \dots \quad (5\cdot2)$$

$$\sin (\omega t - \delta) = \sin \{\omega (t+T) - \delta\}$$

This is true only when

$$\omega(t+T) - \delta = (\omega t - \delta) + 2\pi$$

so that
$$\omega T = 2\pi$$

$$\omega = \frac{2\pi}{T} \qquad \dots \quad (5\cdot3)$$

**The frequency** of a vibrating particle is the number of complete oscillations executed by it per second. Hence.

$$\text{frequency} = \nu = \frac{1}{T},$$

$$\therefore \qquad \omega = 2\pi\nu \qquad \dots \quad (5\cdot4)$$

Equations (4.1) may therefore be written as

$$y = a \sin (2\pi\nu t - \delta) \qquad \dots \quad (5\cdot5)$$

**Phase of a vibrating particle.** Phase of a vibrating particle at any instant is its state of motion at that instant. We have seen that the displacement, velocity, and acceleration of a vibrating particle change from instant to instant and occur regularly after equal intervals of time called the time-period State of motion of a vibrating particle at any instant is determined by its displacement, velocity, and acceleration at that instant.

We have already seen that the general equation for displacement of a particle executing simple harmonic motion is

$$y = a \sin (\omega t - \delta).$$

The velocity of the particle at the same instant is

$$v = a\omega \cos(\omega t - \delta)$$

The acceleration of the same particle at the same instant is

$$f = -a\omega^2 \sin(\omega t - \delta).$$

Since $(\omega t - \delta)$ is common to all these equations, by knowing $(\omega t - \delta)$, the displacement, velocity and acceleration of a particle executing S.H.M. at an instant $t$ can be determined. It follows that the *mathematical expression for phase of a vibrating particle at an instant t is the angle* $(\omega t - \delta)$.

**Epoch** is the phase of the particle at the instant $t = 0$. Therefore epoch is represented by the angle $\delta$ in equation (4·1)

**6. Energy of a Particle executing S. H. M.** A particle executing simple harmonic motion possesses both kinetic and potential energies. It possesses kinetic energy by virtue of its motion. Its potential energy at any point on its path is due to the work done against the force of restitution to bring the body to the position under consideration. The two energies can be calculated in the following way ·

**The kinetic energy.** Let $v$ be the velocity of the body at a distance $y$ from its position of equilibrim. If $m$ be the mass of the particle, then its kinetic energy is

$$T = \tfrac{1}{2} mv^2$$

$$T = \tfrac{1}{2} ma^2\omega^2 \cos^2(\omega t - \delta) \text{ from Eqn. (4·2)} \quad (6·1)$$

**The potential energy.** Potential energy of the particle is equal to the work done against the force of restitution which is proportional to the displacement $y$ of the particle from the mean position. From Eqn. (4·4) the force of restitution at a distance $y$ is

$$P = -mw^2y$$

The work done in a small additional displacement $dy$ is

$$Pdy = -m\omega^2 ydy.$$

Hence the total work done to displace the body over a distance $y$ is

$$W = \int_0^y Pdy = m\omega^2 \int_0^y ydy = \tfrac{1}{2} m\omega^2 y^2$$

or

$$W = \tfrac{1}{2} m\omega^2 a^2 \sin^2(\omega t - \delta) \quad (6·2)$$

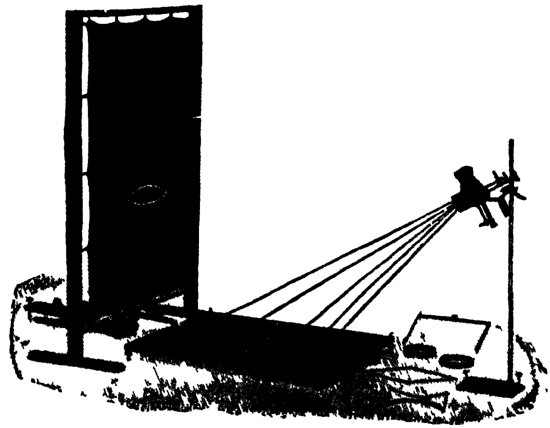


Fig 1.

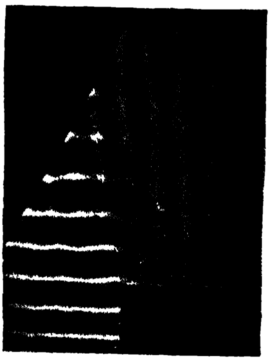


Fig 2

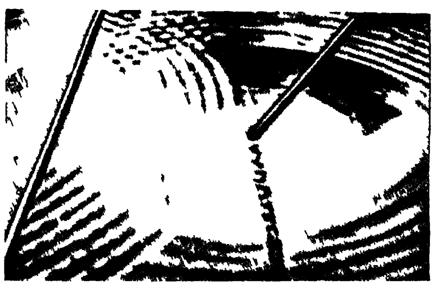


Fig. 3

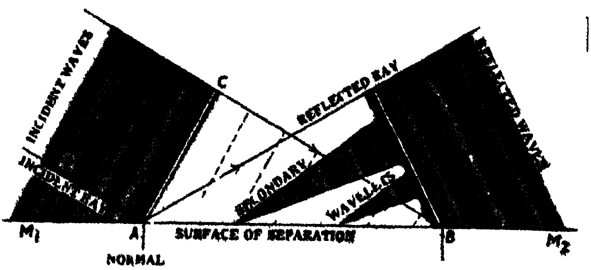



Fig. 4.

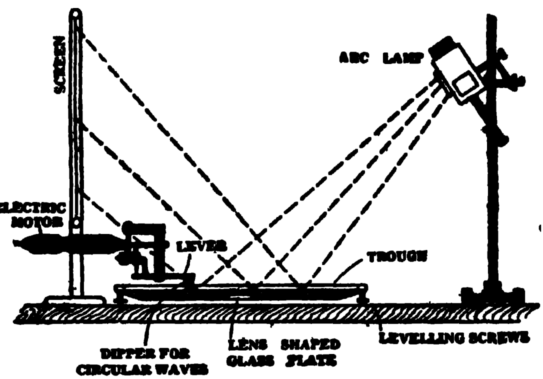
This work remains stored up in the body as its potential energy at a distance $y$.

Hence the total energy of the body at a distance $y$ from its equilibrium position is

$$E=T+W=\frac{1}{2}ma^2\omega^2 \qquad \ldots \quad (6\cdot3)$$

Since the expression for energy is independent of $y$ and $t$, the energy of the oscillating particle is the same at all instants and in all positions. The mechanical energy of the particle can only alternate between kinetic and potential forms.

**7. The Ripple Tank.** As students are very familiar with water waves, we can best illustrate the properties of waves with the help of ripples generated on the surface of water. The essential apparatus required for this purpose is a ripple tank.




Fig, II-1

The ripple tank is a piece of apparatus by which ripples can be generated continually for a long time on the surface of water. The apparatus is shown in Plate 1 (Fig. II-1) and the sectional diagram is shown in Fig. II-1 of the text. It consists of the following parts.:

**The trough*.** This is a shallow rectangular trough with gradually sloping edges running below the water level corresponding to a shelving beach to avoid the effect of reflection

*The tank described here has been designed by Messrs. E. Leybolds Nachfolger. It is very convenient for class demonstration.

of the ripples from the sides of the tank. The trough stands on three levelling screws. At the bottom of the trough there is a plane mirror. This serves two purposes: it reflects much larger quantity of light falling on the trough and also helps to make the depth of water in the trough the same everywhere.

**The rippler.** It is a vibrating lever with arrangement for different attachments called dippers for generating different types of waves. The lever is operated by means of an eccentric cone fixed to the shaft of a small 6-volt direct motor which can be run by a battery of a few dry cells : The frequency and amplitude of vibration of the lever can be altered to any desired extent within a small range.

To set the apparatus, the previously cleansed trough with the glass plate at its bottom is first levelled by means of the levelling screws and a spirit level. The glass plate is then covered with clean water to a depth of few millimetres. The trough is next illuminated by light from a powerful carbon arc placed obliquely at a definite height on one side of the trough. The reflected light is then received on a vertical screen placed on the other side of the trough away from the arc.

To produce circular waves, a small cylindrical rod with slight taper is attached to the end of the lever which is made to vibrate with a large amplitude and with low frequency.

To generate plane waves, the cylindrical dipper for producing circular waves is replaced by a moderately long, flat, and thin plate. On the screen, the circular waves appear as bright concentric circles (rather, ellipses due to inclination of the screen to the reflected beam, see Plate IV, Fig. 6), while the plane waves appear as a series of illuminated parallel lines.

**8. Progressive or Phase Waves.** If we watch the movement of any small floating body (such as a cork particle) on the surface of water traversed by waves, it will be found to move up and down (more precisely, in vertical circles) about its normal position without being carried bodily by the waves in the direction of wave propagation. This shows that when waves move over the surface of water, there is no flow of water

in the direction of wave motion. What is it then that is really moving ? To answer this question we shall make use of the apparatus shown in Fig. II-2.

The apparatus consists of a number of small balls each carried at the end of a rod. Each rod rests on a wheel fitted eccentrically to a shaft that can be turned by a handle. On

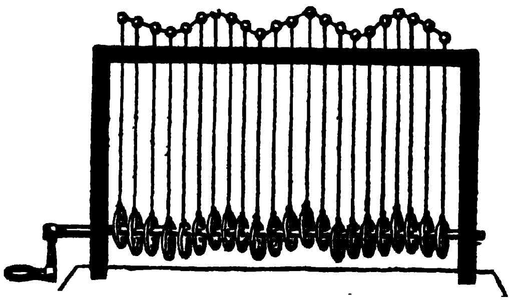


Fig. II-2

turning the handle, the balls move up and down, while the wave-curve represented by the instantaneous positions of these balls appear to move in the forward or backward direction. So *wave-propagation means advancing of a form generated by the instantaneous positions of particles lying in the path of the waves.* During wave propagation, through water (say) water is heaped up in some places, while it is depressed in other places. These elevations above the normal undisturbed surface are called *crests* and the depressions below the normal level are called *troughs* or *hollows*.

**Phase velocity.** We can also look at the mechanism of wave propagation from a different angle of vision. Each particle taking part in the propagation of waves moves up and down about its mean position in a periodic manner as the waves sweep over them, and the phase of motion of the particles at any instant changes from particle to particle along the line of wave propagation. Nevertheless, the phase of a particle at B (see Fig. II-3) at any instant becomes the same as that

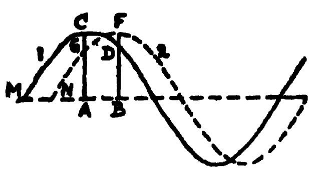



Fig. II-3

**Condition 4.** Again, let A and B be two nearest particles in a wave train which are in the same phase of vibration at an instant $t$. Then, if $x_1$ be the co-ordinate of A, that of B must be $(x_1+\lambda)$.

Hence, at any instant $t$, the displacement at A is

$$y_1=a\cos\frac{2\pi}{T}\left(t-\frac{x_1}{C}\right)$$

The displacement at the same instant at B is

$$y_2=a\cos\frac{2\pi}{T}\left(t-\frac{x_1+\lambda}{C}\right)$$

Since the particles A and B are supposed to be in the same phase at the instant $t$, we must have $y_1=y_2$

$$\therefore\qquad \cos\frac{2\pi}{T}\left(t-\frac{x_1}{C}\right)=\cos\frac{2\pi}{T}\left(t-\frac{x_1+\lambda}{C}\right)$$

This is true only when

$$\frac{2\pi}{T}\left(t-\frac{x_1}{C}\right)=\frac{2\pi}{T}\left(t-\frac{x_1+\lambda}{C}\right)+2\pi,$$

since the particles A and B are different.

or $$t-\frac{x_1}{C}=t-\frac{x_1+\lambda}{C}+T$$

or, $$\frac{\lambda}{C}=T$$

$$C=\frac{\lambda}{T}=\nu\lambda \qquad \text{...(10·2)}$$

Thus, Eqn. (10·1) satisfies all the conditions of propagation of waves. It is, therefore, the mathematical expression for progressive waves. This equation represents propagation of waves in the direction of increasing $x$. The propagation of waves in the direction of diminishing $x$ is represented by

$$y=a\cos\frac{2\pi}{T}\left(t+\frac{x}{c}\right) \qquad \text{...(10.3)}$$

**Forms of wave equation.** The equation of progressive wave can be written in different forms with help of the relations $\nu=1/T$ and $C=\nu\lambda$. Thus

$$y=a\cos\frac{2\pi}{T}\left(t-\frac{x}{C}\right)$$

$$=a\cos\frac{2\pi C}{\lambda}\left(t-\frac{x}{C}\right)$$

$$=a\cos\frac{2\pi}{\lambda}(Ct-x) \quad \ldots \quad (10\cdot4)$$

$$=a\cos\frac{2\pi}{\lambda}(x-Ct) \quad \ldots \quad (10\cdot5)$$

$$=a\cos 2\pi\left(\frac{x}{\lambda}-\frac{Ct}{\lambda}\right) \quad \ldots \quad (10\cdot6)$$

$$=a\cos 2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right) \quad \ldots \quad (10\cdot7)$$

**The phase.** In equation (10·5) the phase of a particle at $x$ at instant $t$ is represented by $2\pi(x-Ct)/\lambda$. The difference of phase between any two points of a medium traversed by waves at distances $x_1$ and $x_2$ from the origin at an instant $t$ is

$$\delta=\frac{2\pi}{\lambda}(x_1-Ct)-\frac{2\pi}{\lambda}(x_2-Ct)$$

or $$\delta=\frac{2\pi}{\lambda}(x_1-x_2) \quad \ldots \quad (10\cdot8)$$

Thus. **Phase difference** $=\frac{2\pi}{\lambda}\times$ **(Path difference)**

So long as waves move in a medium which is homogeneous throughout, they move with uniform velocity and their form remains unaltered. But, in moving from one homogeneous medium to another, their velocity and wave-length are changed. We shall now study the changes produced in form and velocity

of propagation of waves when they move from one homogeneous medium to another.

**11. Characteristics of Progressive Waves.**

(A) **Waves can be reflected.** When waves move from one homogeneous medium into another, at the surface of separation, they are partly reflected into the original medium and partly transmitted into the other. *When the second medium is denser than the first, a crest or a trough is reflected as such, but, if the second medium is rarer than the first, a crest is reflected as a trough and vice versa.* We shall first study the nature of the reflected waves under different conditions.

**Reflection of plane waves at a plane surface.** To study the behaviour of waves under this circumstance, we use the ripple tank and generate waves in the manner already described. The photograph in Fig. 2 (Plate I) shows both the incident and the reflected waves from a rigid barrier. Since, by reflection, some energy is lost, the reflected waves are much fainter than the incident waves. It will be seen that the plane waves after reflection at a plane surface remain plane but they are turned through a certain angle.

It will be noticed from Fig. 2 (Plate I) that the incident and reflected waves are equally inclined to the reflector. Hence, the incident and the reflected rays are equally inclined to the normal to the reflecting surface.

**Reflection of spherical waves from a plane surface.** Spherical waves diverging from a point, when reflected from a plane surface, remain spherical (Fig. 3. Plate I) and appear to diverge from a centre lying behind the reflector. This centre is called the virtual centre. Since, the incident and the reflected waves move with the same velocity, the virtual centre lies as far behind the reflector as the real centre is in front of it.

**Reflection of plane waves from a spherical surface.** The spherical reflector may either be concave or convex towards the source. In the former case (Fig. I, Plate II), plane waves, after reflection, become spherical and converge to a point in front of the reflector at a distance equal to half the radius of the reflector. This point is called the *focus*. Conversely, spherical

PLATE II

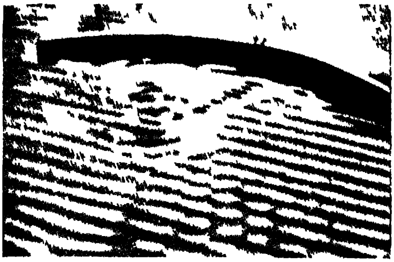


Fig 1.

Fig. 2

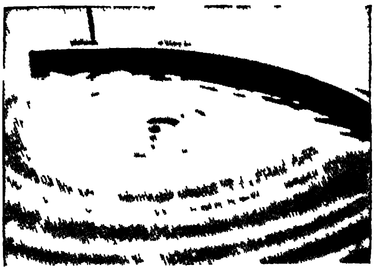


Fig. 3

Fig. 4.

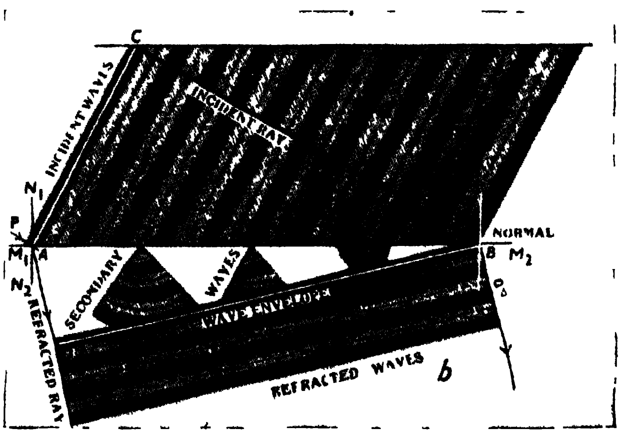



Fig. 5.

To face P. 34.

waves diverging from the focus become plane after reflection at a concave spherical surface.

In the latter case, these waves appear to diverge from a point behind the convex reflector, the centre lying at a distance equal to half the radius of the reflector.

**Reflection of spherical waves from a spherical surface.** Spherical waves reflected from a spherical surface are also spherical (Figs. 2 and 3, Plate II) and the position of centre of the reflected waves depends on the position of the source and the radius of the reflector.

**Reflection of plane waves from a parabolic reflector.** If plane waves are reflected from a parabolic reflector, they become spherical and converge to the geometrical focus of the parabola. Conversely, if the source of spherical waves be placed at the focus of the parabola, the reflected waves become plane.

**(B) Waves can be refracted.**

To study the phenomena of refraction of waves, a very ingenious device has been used in the type of ripple tank described above. We have already mentioned that in moving from one medium to another, the velocity of propagation of waves changes. It is this change in velocity which causes the waves to bend (or to be refracted) on meeting the surface of separation between the two different media. Now it is well-known that *velocity of water waves depends on the depth of water.* Hence so far as the effect of bending of waves due to change in velocity alone is concerned, the change in homogeneity of a liquid medium (here water) is equivalent to a change in depth of the liquid. In the ripple tank under consideration the depth of the liquid is changed by means of a plate glass immersed under water in the tank. The plate glass may be of the shape of a lens or a prism. (See Fig. 1, Plate I.

How this bending of waves due to change in velocity occurs can be followed from the following simple experiment. Cover half the surface of a table by felt and the other half by a glass plate, the two halves meeting one another along a line. If we allow a glass rod to roll over the table obliquely to the line separating

glass from felt, it will move with constant velocity so long as the whole length of the rod lies on felt. On emerging from the felt, the portion of the rod in contact with glass moves faster than the portion lying on felt. This causes the rod to rotate about the line of separation, so that, when the rod leaves the felt completely it becomes inclined to the line of separation at a different angle.

To exhibit the phenomenon of refraction of plane waves at a plane surface, a plane mirror with parallel faces is immersed under water in the trough, so that, the depth of water above the mirror is less than that outside it. When plane waves pass through water above the mirror, the direction of propagation of the waves is changed (Fig. 4 Plate II). But on emerging from the mirror, it again becomes parallel to the original direction. Note that the wave length above the glass plate is shorter than that outside.

**Refraction of plane waves through a prism.** To study the phenomenon of refraction through a prism, a mirror of the shape of double prism is immersed under water in the trough (Fig. 1, Plate III). It will be noticed that plane waves after refraction through the prism remain plane but they are inclined towards their common base.

**Refraction through a lens.** A lens shaped medium has the property of impressing curvature on a train of plane waves. If the lens is convex, or thickest in the middle (Fig. 2, Plate III), the centre of these emergent spherical waves lie on the other side of the lens away from the source of disturbance. If the lens is concave, or thinnest in the middle (Fig. 3, Plate III) the centre of the divergent waves, on emergence, lies on the same side of the lens as the source. If the incident waves are spherical, the emergent waves from the lens are generally spherical but the curvature is altered. (Fig. 4, Plate III)

(C) **Interference of Waves** Two trains of waves of different wave lengths, frequencies and amplitudes moving over the same part of a medium interfere with each other. In this case the amplitudes of two component waves are superposed and there is only one resultant wave. This is what is to be expected, for any given particle of the medium cannot be simultaneously displaced in two different directions. In

PLATE III

Fig 1

Fig 2

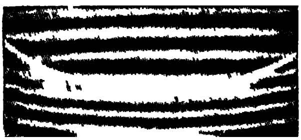


Fig 3

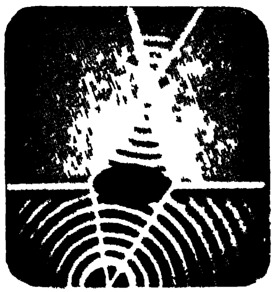


Fig 4

Fig 5

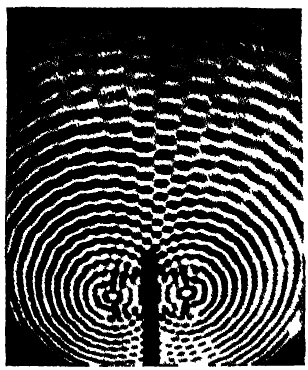


Fig 6

*To face P 36*

the process of combination where a crest meets another crest (rather, where two waves meet in the same phase) the displacements are added up, but where a crest meets a trough (or where the two waves meet in opposite phases) the resultant displacement is obtained by taking the difference of displacements due to the two component waves. This principle of combination of displacements due to component waves to obtain a resultant wave is called the *principle of superposition.* This effect is beautifully illustrated by means of the ripple tank alrady described. In this case the dipper consists of two vibrating pins fixed to the ends of a rod and moved up and down simultaneously (in the same phase) by means of the vibrating lever to which the rod is screwed. The two pins generate two similar trains of circular waves which by their criss-crossing, interfere with each other. This is shown in Fig. 6, Plate III. It is to be noted that along two systems of hyperbolic branches (which have their foci at the two sources) there is no displacement at all. It is along these lines that the two trains of waves always meet in opposite phases, and, neutralise the effect of each other. In between these systems of hyperbolas, there is another system where the two trains of waves always meet in the same phase and, therefore, add up their effects.

(1) **Stationary waves** This is a train of waves which does not appear to progress or move in the medium and are stationary in space. They are formed when two trains of progressive waves of equal amplitude and wave length move over the same part of the medium with equal velocities and in opposite directions. This important case of interference can be illustrated by the following experiments.

**Experiment 1.** Take some quantity of water in a rectangular tin-can and gently tap one of the sides of the can at suitable intervals. The water surface will show a chequerred appearance which will remain stationary. This is due to interference of incident waves and those reflected from the sides of the trough.

**Experiment 2.** The properties o stationary waves can be best studied with the help of the Melde's apparatus shown in Fig.II-6. It consists of a tuning fork fixed to the base board of

the instrument in an upright position. One end of a string is fixed to one prong of the tuning fork and the other end carries a scale pan after passing over a frictionless pulley. To perform the experiment, suitable weights are placed on the scale pan and the tuning fork is excited by means of a violin bow. Then stationary waves are formed by superposition of incident waves generated by the fork and the waves reflected from the pulley-end of the string. The following features of stationary waves are worth noting :

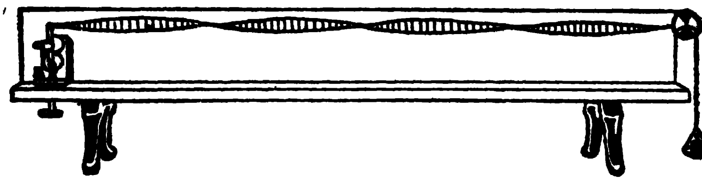


Fig. II-6

(*1*) *The medium (in this case, the whole length of the string) is divided into a number of segments or* **loops** *of equal length separated by points which have no motion at all. These points are called* **nodes.**

(*2*) *The mid-points of these segments have maximum amplitude of vibration. These are called* **antinodes.** *The amplitudes of vibration of particles lying on either side of the antinodes gradually fall off till they become zero at the nodes.*

(*3*) *All the particles lying in one loop always vibrate in the same phase; thus, all of them acquire their maximum displacements on the same side of the string or pass through their normal positions in the same direction at the same instant. But the corresponding particles on the opposite sides of a node are always in opposite phases.*

*The distance between two consecutive nodes or consecutive antinodes is half the wave length of the component waves.*

**Mathematical treatment of stationary waves.** We have seen that the displacement of a particle in a medium at an

instant $t$ and at a distance $x$ from the source of disturbance due to waves moving in the direction of increasing $x$ is given by

$$y_1=a\cos\frac{2\pi}{\lambda}(x-Ct)$$

and that due to a similar wave moving in the opposite direction with the same velocity is given by

$$y_2=a\cos\frac{2\pi}{\lambda}(x+Ct)$$

Hence the displacement of the particle at $x$ at instant $t$ due to the two waves moving simultaneously through the medium in opposite directions is

$$\begin{aligned} y&=y_1+y_2\\ &=a\left[\cos\frac{2\pi}{\lambda}(x-Ct)+\cos\frac{2\pi}{\lambda}(x+Ct)\right]\end{aligned}$$

or $$y=2a\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi Ct}{\lambda}$$

or $$y=A\cos\frac{2\pi Ct}{\lambda} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11\cdot1)$$

where $$A=2a\cos\frac{2\pi x}{\lambda} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11\cdot2)$$

Here $A$ is the amplitude of the particle at $x$ at instant $t$.

The Eqn. (11.1) is expressed as the product of two terms one of which involves $x$ only and the other time $t$ only. In this expression $A$ is the amplitude factor and the other is the phase factor. Hence phase is independent of $x$ and, therefore, it is the same at all points at any instant in a single loop.

The amplitude $A$ is zero, whatever the value of $t$, if

$$\cos\frac{2\pi x}{\lambda}=0$$

$$\frac{2\pi x}{\lambda}=(2s+1)\frac{\pi}{2} \quad \text{where } s=0,\ 1,\ 2,\ 3,\ldots\ldots$$

or $$x=(2s+1)\frac{\lambda}{4}$$

Hence, corresponding to $s=0$ $\quad x_0=\frac{\lambda}{4}$

" " $s=1$, $\quad x_1=\frac{3\lambda}{4}$

" " $s=2$. $\quad x_2=\frac{5\lambda}{4}$

.. .. etc. etc. etc.

Hence, at distances $x_0$, $x_1$, $x_2$, $x_3$,...the amplitude is permanently zero. These are the positions of the nodes. The distance between two consecutive nodes is, therefore,

$$x_2 - x_1 = \frac{5\lambda}{4} - \frac{3\lambda}{4} = \frac{\lambda}{2}$$

Hence the distance between two consecutive nodes is half the wave length of the component waves. It can similarly be shown that the distance between two consecutive antinodes is also half the wave-length of the component waves.

(D) **Waves can be diffracted.** If an obstacle is placed in the path traversed by waves, there is no disturbance behind the obstacle. This region of no disturbance is called the shadow of the waves. The region bounded by the lines joining the source to the edges of the obstacle is called the *geometrical shadow of the obstacle.* Figures 1, 2, 3, and 4 of Plate IV give the nature of shadows cast by obstacles and apertures of different sizes. It will be noticed that these shadows have no sharp boundaries and that some waves penetrate into the region of geometrical shadows bounded by the dotted lines in these figures.

In this connection the special features of the shadow shown in Fig. 5 (Plate III) should not be overlooked. This figure shows that plane waves, on emerging through a very narrow aperture, become spherical round the aperture as centre. In terms of rays this would mean that a bundle of parallel rays, after emerging from a slit, becomes divergent, the amount of divergence increasing as the width of the aperture diminishes. This bending of rays round the edges of an obstacle is called **diffraction.** It would appear from Fig. 5 (Plate III) that a narrow aperture may be regarded as a secondary source of spherical waves. This statement is further borne out by Fig. 5 (Plate IV). This figure shows three narrow slits placed side by side in the path of a train of plane waves. The interference pattern produced by the three secondary sources thus formed is similar to that formed by three primary (or real) sources of disturbance.

PLATE IV

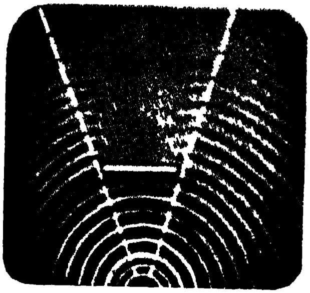


Fig 1

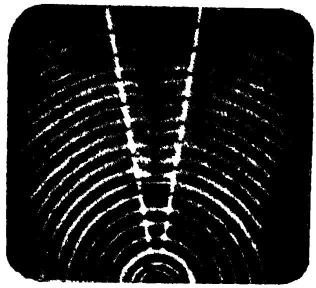


Fig 2

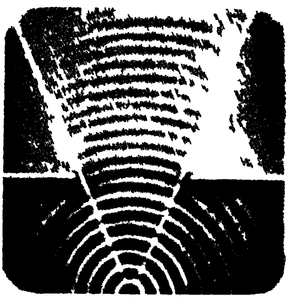


Fig 3

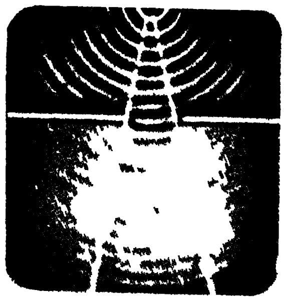


Fig 4

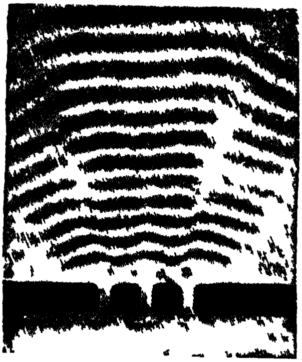


Fig 5


Fig 6.

*To face P. 40.*

**12. Longitudinal Waves.** The waves so far studied are called transverse, because, in the case of waves on water, the particles constituting the medium move perpendicularly to the direction of propagation of waves. It may be remarked here that only those media can transmit transverse waves which possess elasticity of shear, for, in the case of transverse wave propagation the different layers of the medium move tangentially with respect to each other.

We have yet to describe another type of waves where the direction of vibration of the particles are parallel to the direction of wave propagation. These waves are, therefore, called longitudinal waves. During propagation of longitudinal waves each layer of the medium is alternately compressed and rarefied. Hence for the propagation of longitudinal waves alone, the medium must possess bulk elasticity only. Gases do not possess shear elasticity but they possess bulk elasticity. Hence, through gases, only longitudinal waves can be propagated.

Leaving aside this distinction regarding the mode of vibration of particles of a medium traversed by transverse and longitudinal waves, the physical characteristics of these two types of waves are essentially similar; that is, both these types of waves can be reflected, refracted, diffracted and made to interfere with each other according to the principle of superposition.

**13. Flux of Energy.** When waves pass through an elastic medium, there is no mass movement of particles of the medium in the direction of motion. Each particle of the medium oscillates about its mean position. Accordingly, every particle of the medium possesses certain amount of energy which changes from kinetic to potential and from potential to kinetic. In this case the source of disturbance loses energy and it is carried from particle to particle through the medium by means of waves, so that, waves may be regarded as vehicles of energy.

The flux or flow of energy through the medium may be measured by the quantity of energy flowing through a unit cross-section (per square centimetre) placed perpendicularly to the direction of propagation. Hence it is equal to energy

contained in a cylinder of unit cross-section and of length equal to the magnitude of velocity of propagation $C$ of the waves; in other words, it is equal to the energy contained in a volume of $C$ cubic centimetres.

We have already calculated the total energy of a particle of mass $m$ and oscillating with a period given by $\omega$ ($=2\pi/T$.) It is equal to $\frac{1}{2}ma^2\omega^2$, $a$ being the amplitude of oscillation. If there be $n$ particles of the medium per unit volume, then the energy per unit volume is $\frac{1}{2}mna^2\omega^2$. Hence the flux of energy is

$$J=\tfrac{1}{2}mnCa^2\omega^2$$

Since $mn=\rho$, the density of the medium, the flux of energy is

$$J=\tfrac{1}{2}\rho Ca^2\omega^2 \qquad \ldots \quad (13{\cdot}1)$$

Since $\omega=2\pi/T=2\pi\nu$, where $\nu$ is the frequency

$$J=2\pi^2\rho Ca^2\nu^2 \qquad \ldots \quad (13{\cdot}2)$$

or, since $C=\nu\lambda$

$$J=2\pi^2\rho\lambda a^2\nu^3 \qquad \ldots \quad (13{\cdot}3)$$

In any case the *flux of energy of a train of waves of given frequency the intensity is proportional to the square of the amplitude of vibration. Also for a disturbance of given amplitude the intensity is proportional to the square of frequency.*

# APPENDIX TO CHAPTER II

**Composition of two simple harmonic motions of the same frequency executed in the same line.** Let $x_1$ and $x_2$ be the two displacements of a particle executing simple harmonic motions an instant $t$. Then, if $\omega$ is the circular-frequency *which is supposed to be the same for both the oscillations,* we have

$$x_1 = a_1 \cos(\omega t - \alpha_1) \qquad \ldots \quad (1)$$

$$x_2 = a_2 \cos(\omega t - \alpha_2) \qquad \ldots \quad (2)$$

where $a_1, a_2$ and $\alpha_1, \alpha_2$ are the respective amplitudes and epochs of the two oscillations. The resultant displacement of the particle is evidently,

$$x = x_1 + x_2 = a_1 \cos(\omega t + \alpha_1) + a_2 \cos(\omega t + \alpha_2)$$

or,
$$x = a_1(\cos \omega t \cos \alpha_1 - \sin \omega t \sin \alpha_1) + a_2(\cos \omega t \cos \alpha_2 - \sin \omega t.\ \sin a_2)$$

or,
$$x = \cos \omega t\,(a_1 \cos \alpha_1 + a_2 \cos \alpha_2) - \sin \omega t(a_1 \sin \alpha_1 + a_2 \sin \alpha_2) \qquad \ldots \quad (3)$$

Putting $a \cos \delta = a_1 \cos \alpha_1 + a_2 \cos \alpha_2$
and $a \sin \delta = a_1 \sin \alpha_1 + a_2 \sin \alpha_2$

so that
$$a^2 = a_1^2 + a_2^2 + 2a_1a_2 (\cos \alpha_1 \cos \alpha_2 + \sin \alpha_1 \sin \alpha_2)$$

or,
$$a^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos(\alpha_1 - \alpha_2) \qquad \ldots \quad (4)$$

and
$$\tan \delta = \frac{a_1 \sin \alpha_1 + a_2 \sin \alpha_2}{a_1 \cos \alpha_1 + a_2 \cos \alpha_2} \qquad \ldots \quad (5)$$

We get from Eqn. (3) $x = a(\cos \omega t \cos \delta - \sin \omega t.\ \sin \delta)$

$$x = a \cos(\omega t + \delta) \qquad \ldots \quad (6)$$

It follows that the resultant vibration of the particle is also simple harmonic, having the same frequency as that of the components but with different amplitude and phase.

**Geometrical method of composition of simple harmonic motions.** Eqns. (4) and (5) lead to the following *geometrical method of composition of two simple harmonic motions of the same frequency*: Draw a line OX (Fig. 1). From the point O draw a line OA of length proportional to $a_1$ and making an angle $\alpha_1$ with OX. From the point A draw another line AB of length proportional to $a_2$ and making an angle $\alpha_2$ with the direction of OX. Complete the parallelogram OABC with OA and AB as adjacent sides. Then the diagonal OB is proportional to the resultant amplitude and the angle which OB makes with OX gives the phase of the resultant.

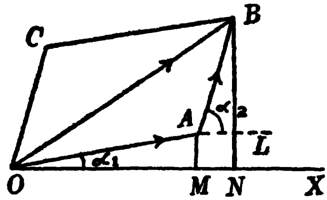


Fig. 1.

**Proof.** To prove this, let BN and AM be the perpendiculars from B and A on OX. From A drop a perpendicular AL on BN. Then

$$OB^2 = OA^2 + OC^2 + 2.OA.OC.\cos\angle AOC$$
$$= a_1^2 + a_2^2 + 2a_1a_2\cos(\alpha_2 - \alpha_1),$$

Comparing with Eqn. (4), we find that OB is equal to the magnitude of the amplitude $a$ of the resultant vibration. Again,

$$\tan BOX = \frac{BN}{ON} = \frac{BL+LN}{OM+MN} = \frac{BL+AM}{OM+AL}$$

$$\text{or,}\quad \tan BOX = \frac{a_2\sin\alpha_2 + a_1\sin\alpha_1}{a_2\cos\alpha_2 + a_1\cos\alpha_1}.$$

Comparing with Eqn. (5), we find that $\angle BOX = \delta$, the phase of the resultant vibration.

**Extension of the geometrical method.** The above method of composition of two simple harmonic motions can be easily extended to any number of simple harmonic motions of the same period but of different amplitudes and phases in the same way that any number of forces can be compounded

together by the polygon method. Thus, let $(a_1, \alpha_1)$, $(a_2, \alpha_2)$, $(a_3, \alpha_3)$, $(a_4, \alpha_4)$ etc. be the respective amplitudes and phases of the *first*, *second*, *third*, *fourth* etc. simple harmonic motions in the

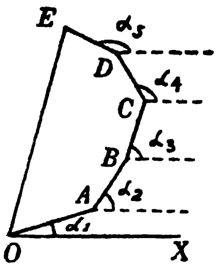



Fig. 2

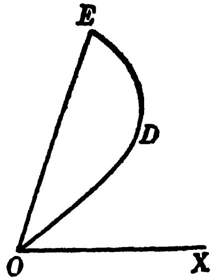



Fig. 3

same line, impressed on a particle. To obtain the resultant amplitude and phase, we draw a line OX as in Fig. 2. From the point O we draw a line OA of length proportional to $a_1$ and inclined at angle $\alpha_1$ to OX; from A we draw a line AB of length proportional to $a_2$ and inclined at angle $\alpha_2$ to OX. We proceed in the same way till the last amplitude has been drawn. The line OE joining O to the end point E of the last amplitude gives the resultant amplitude. The angle of inclination of OE to OX gives the phase of the resultant.

If the amplitudes are small and phases change by infinitesimaly small amount, then the polygon becomes a continuous curve (Fig. 3). We shall refer to this curve as **vibration curve.** As a particular case if the successive amplitudes are small but equal and the phases change by equal small amounts, then the vibration curve becomes arc of a circle.

**Composition of two rectangular simple harmonic motions.** Let the two vibrations of equal frequencies but of different amplitudes and phases be impressed on the same particle in two perpendicular directions OX any OY. Then the particle would describe a curve whose orbit depends on the period, phase, and relative amplitudes of vibration of the two particles.

Let the equations of motion of these two vibrations be

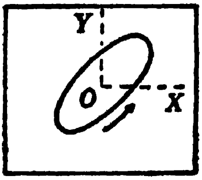


Fig. 4

$$x = a \cos \omega t \qquad \ldots \quad (7)$$
$$y = b \cos (\omega t + \delta) \qquad \ldots \quad (8)$$

where $a$ and $b$ are the amplitudes of vibration along OX and OY, and $\delta$ is the difference of phase. To find the orbit of the particle, we have to eliminate $t$ between the two equations thus : From Eqn. (7)

$$\cos \omega t = \frac{x}{a}, \text{ so that, } \sin \omega t = \sqrt{1 - \cos^2 \omega t} = \sqrt{1 - \frac{x^2}{a^2}}.$$

Substituting for $\cos \omega t$ and $\sin \omega t$ in Eqn. (8)

$$\frac{y}{b} = \cos \omega t \cos \delta - \sin \omega t \sin \delta$$

we get, $$\frac{y}{b} = \frac{x}{a} \cos \delta - \sqrt{1 - \frac{x^2}{a^2}} \sin \delta$$

or $$\left(\frac{y}{b} - \frac{x}{a} \cos \delta\right)^2 = \left(1 - \frac{x^2}{a^2}\right) \sin^2 \delta$$

or $$\frac{y^2}{b^2} + \frac{x^2}{a^2} \cos^2 \delta - \frac{2xy}{ab} \cos \delta = \sin^2 \delta - \frac{x^2}{a^2} \sin^2 \delta$$

or $$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} \cos \delta = \sin^2 \delta \qquad (9)$$

Eqn. (9) represents the general equation of an ellipse whose centre coincides with the origin of co-ordinates O (Fig. 4). We shall here consider some particular cases of Eqn. (9).

**Case 1.** If the phase difference $\delta = 0$, so that, $\sin \delta = 0$ and $\cos \delta = 1$, the Eqn. (9) transforms into

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

or $$\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$$

or $$y = \frac{bx}{a}. \qquad \ldots \quad (10)$$

Eqn. (10) represents the equation of a straight line passing through the origin O and inclined to the positive direction of OX at an angle whose tangent is $\frac{b}{a}$. Thus, the orbit of the oscillating particle is a straight line inclined to the X-axis (Fig. 5).

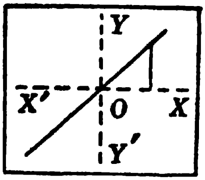


Fig. 5

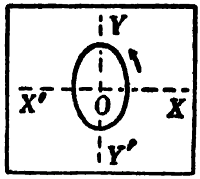


Fig. 6

**Case 2.** If the phase difference $\delta=\pi/2$, so that, $\sin\delta=1$, and $\cos\delta=0$, then, the Eqn. (9) takes the form

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \dots \quad (11)$$

Eqn. (11) represents the equation of an ellipse whose semi-axes coincide with the X-and Y-axes (Fig. 6).

If, in addition to $\delta$ being equal to $\pi/2$, the amplitudes $a$ and $b$ are equal, then, Eqn. (11) becomes

$$x^2+y^2=a^2. \qquad \dots \quad (12)$$

In other words, the orbit becomes a circle whose centre coincides with the origin of co-ordinates O (Fig. 7).

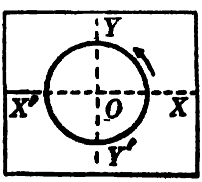


Fig. 7

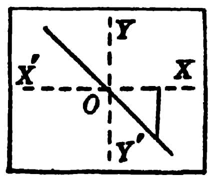


Fig. 8

**Case 3.** If $\delta=\pi$, so that, $\sin\delta=0$, and $\cos\delta=-1$, then, Eqn. (9) reduces to

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{2xy}{ab}=0$$

or
$$\left(\frac{x}{a}+\frac{y}{b}\right)^2=0$$

or
$$y=-\frac{b}{a}.x \qquad (13)$$

In this case the orbit of the particle is again a straight line passing through the origin and making an obtuse angle with the positive direction of the X-axis (Fig. 8).

*Corollary.* Two simple harmonic motions represented by

$$\left.\begin{array}{l} x=a\cos\omega t \\ y=a\sin\omega t \end{array}\right\} \qquad \dots \quad (14)$$

represent a circular vibration, since by squaring these equations and adding them, we get

$$x^2+y^2=a^2.$$

If we change the sign of $\omega$, then the $x$ and $y$-vibrations take the form

and
$$\left.\begin{array}{l} x=a\cos\omega t \\ y=-a\sin\omega t. \end{array}\right\} \qquad \dots \quad (15)$$

These also represent a circular vibration, since by squaring and adding they yield

$$x^2+y^2=a^2$$

But, in the latter case (Eqns. 15) the circular orbit is described in the opposite sense as compared to the former case (Eqns. 14), since the sign of $\omega$ is negative here.

Conversely, if we superpose these two circular motions, we have to add up the $x$-components and the $y$-components separately in Eqns. (14) and (15) thus obtaining,

and
$$\left.\begin{array}{l} x'=2x=2a\cos\omega t \\ y'=0 \end{array}\right\} \qquad \dots \quad (16)$$

Eqn. (16) represents the Equation of simple harmonic motion along a straight line. It follows that a simple harmonic motion along a straight line represented by

$$x=a\cos\omega t$$

can be regarded as superposition of two circular motions of half the amplitude and of the same period but described in opposite senses and represented by

$$\left.\begin{array}{l} x=\tfrac{1}{2}a\cos\omega t \\ y=\tfrac{1}{2}a\sin\omega t \end{array}\right\} \text{and} \qquad \left.\begin{array}{l} x=\tfrac{1}{2}a\cos\omega t \\ y=-\tfrac{1}{2}a\sin\omega t \end{array}\right\}$$

# CHAPTER III

## HUYGEN'S' PRINCIPLE AND GEOMETRICAL OPTICS

**Recapitulation.** We have discussed in the introductory chapter the merits and demerits of the two possible theories of light namely, the corpuscular theory and the wave theory or the elastic-solid theory. The corpuscular theory is not capable of explaining all the observed phenomena and requires to be supplemented by additional assumptions, such as theory of fits, leading sometimes to results which are contrary to experience. Accordingly, corpuscular theory was abandoned.

The wave theory not only explains most of the observed phenomena correctly but also predicts phenomena which have been actually observed. According to this theory the whole space is pervaded by a hypothetical elastic fluid called *the ether* in which particles of matter are embedded like bodies floating in jelly. Any disturbance created anywhere in this medium is propagated in all directions with the velocity of light in the shape of waves like those moving over the surface of water agitated by the dropping of a stone.

The velocity of propagation of waves in an elastic medium is

$$C=\sqrt{\frac{E}{D}}$$

where $E$ is the elasticity and $D$ is the density of the medium. If follows that the elasticity of the ether must be very high and that its density must be inconceivably low in order that light may be propagated through this medium with as high a velocity as $3\times10^{10}$ cm. per second. No known medium of our experience possesses this property and all attempts to prove the existence of this substance have only produced null results. Another difficulty with the elastic solid theory is that the motion of heavenly bodies through the ether must be impeded, but no certain evidence regarding such impedance has ever been obtained.

Instead of pushing the matter further, we shall here try to determine how far the elastic-solid theory is capable of

explaining the facts of our experience, particularly the laws of geometrical optics. We have seen in the introductory chapter that while the wave theory of light accounts for the finite velocity of light, Huygens' conception of the mode of wave propagation provides the necessary basis on which the mathematical structure of the wave theory of light can be built up.

**Huygens' principle of secondary wavelets.** This principle describes a new mechanism of propagation of waves. According to this principle *each point of an elastic medium occupied by any particular wave-front is the source of secondary wavelets which combine together along an envelope to form the new wave-front.*

Huygens' principle as stated here is not free from objections. We shall see later how this principle is to be modified in order to meet these objections and to explain larger classes of phenomena connected with wave optics.

The laws of reflection, refraction, and also rectilinear propagation of light follow as necessary consequences of Huygens' principle (see Introductory Chapter). It has also been seen how the phenomenon of critical reflection can be readily explained by the same principle. We shall open this chapter by taking up the case of refraction of waves through a prism.

**14. Refraction through Prism.** We shall here consider the case of refraction of plane waves through a prism. On comparing Fig. 1, Plate III, we find that a plane wave on refraction into and after emergence from a prism remains plane and the emergent wave-front becomes inclined to the incident wave-front.

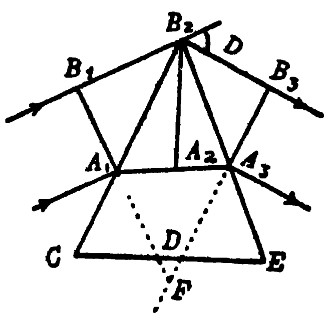


Fig. III-1

In Fig. III-1, the incident wave-front $A_1B_1$ inclined at an angle $\phi$ to the first refracting surface $B_2C$ becomes the wave-front $A_2B_2$ after refraction into the prism. It is inclined to the surface $B_2C$ at angle $\phi'$ and to the surface $B_2E$ at angle $\psi'$. Let the emergent wave-front $A_3B_3$ be

inclined to the surface $B_2E$ at angle $\psi$. It is at once evident from the figure that the refracting angle of the prism is

$$i=\phi'+\psi' \quad .. \quad (14\cdot1)$$

The deviation $D$ between the incident and the emergent rays is equal to the angle between the incident and the emergent wave-fronts, since the rays are normal to the wave-fronts. Since the angle $B_1B_2A_1$ is equal to $(90-\phi)$ and angle $B_3B_2A_3$ is equal to $(90-\psi)$, we have

$$(90°-\phi)+i+(90°-\psi)+D=180°$$

or,
$$D=\phi+\psi-i \quad .. \quad (14\cdot2)$$

Since a wave-front is the locus of all points on a surface which are in the same phase of vibration, and, since phase difference is proportional to path difference (see Eqn. 10·8), it follows that the optical path joining any two corresponding points in the incident and emergent wave-fronts must be the same for all corresponding pairs of points. Hence taking the extreme paths $B_1B_2B_3$ and $A_1A_3$, we have

$$B_1B_2+B_2B_3=n(A_1A_2+A_2A_3)$$

or, $A_1B_2 \sin\phi+A_3B_2 \sin\psi=n(A_1B_2 \sin\phi'+A_3B_2 \sin\psi')$

or, $A_1B_2 (\sin\phi-n \sin\phi')+A_3B_2 (\sin\psi-n \sin\psi')=0$

This is true only when

$$\sin\phi=n \sin\phi' \quad .. \quad (14\cdot3)$$

and
$$\sin\psi=n \sin\psi' \quad .. \quad (14\cdot4)$$

Equations (14·3) and (14·4) give the conditions of refraction at the two refracting faces of the prism $B_2CE$.

**15. Colour and Dispersion.** Light travels with different velocities in different material media. Further, since refractive index of a medium is different for different colours, rays of different colours move with different velocities in a transparent medium. But, in free space rays of all colours move with the same velocity. The evidence of this is furnished by the variable star Algol which would show a sequence of colours as the brightness of the star changes, if rays of different colours travelled through free space with different velocities. For the same reason the satellite of a planet would also show a sequence of colours when it appears or disappears during an eclipse. But this is contrary to experience. It is also an experimental fact that colour of light does not change

on refraction into a transparent medium. We are thus forced to the conclusion that the characteristic of light which we call colour must be connected with frequency of light waves. Hence, in the equation $C=\nu\lambda$ it is the wave-length $\lambda$ that changes with velocity when light passes from one medium into another. This happens because in the case of forced vibration, the frequeney of a driven body is always equal to that of the driving body. If $C_1$, $\lambda_1$ and $C_2$, $\lambda_2$ represent velocity and wave-length in media (1) and (2) respectively, we must have

$$C_1=\nu\lambda_1$$

and

$$C_2=\nu\lambda_2$$

or,

$$\frac{C_1}{C_2}=\frac{n_2}{n_1}=\frac{\lambda_1}{\lambda_2} \quad .. \quad (15.1)$$

By Eqn. (15·1), the wave length of light determined experimentally in any medium can be reduced to that in vacuo, if one of the media (1) or (2) is vacuum.

**16. Reflection and Refraction of Spherical Waves at Plane and Spherical Boundaries.** It will be seen from Figures in Plates II and III that the general effect of refraction of a wave-front at a spherical boundary is to change the curvature of the wave-front after reflection or refraction. It is possible to deduce the relations between conjugate distances from the knowledge of curvatures of wave-fronts before and after reflection or refraction as shown below. We shall first deduce a mathematical expression for curvature of a curve and also an expression for sagitta. This will be useful for subsequent treatment of the subject of reflection and refraction at spherical boundaries.

**Curvature.** By curvature of a curve at a point we mean the rate of bending of the curve. To be more precise, let the tangents to the curve shown in Fig. III-2, at the points Q and P be inclined to the X-axis at angles $\theta$ and $\theta+\delta\theta$. By rate of bending we mean the rate at which the inclination $\theta$ of the tangent at a point on the curve changes as the point moves from P to a neighbouring position Q. If $s$ is the length of the arc between P and

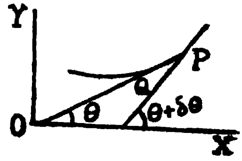



Fig. III-2

Q, then, the rate of bending of the curve, or the average curvature between P and Q is

$$\sigma=\frac{\delta\theta}{\delta s}$$

and the curvature at a point P on the curve is

$$\sigma=\operatorname*{Lim}_{\delta s\to 0}\frac{\delta\theta}{\delta s}=\frac{d\theta}{ds} \qquad (16.1)$$

according to the notation of the differential calculus.

**Curvature of a circle.** In the case of a circle with centre C and radius $R$ (Fig. III-3), let P and Q be two points on the circumference very close to each other. Let the tangents at P and Q make angles $\theta$ and $\theta+\delta\theta$ with the positive direction of the X-axis. Join CP and CQ. Then the angle PCQ is equal to $\delta\theta=\mathrm{PQ}/R$. Hence

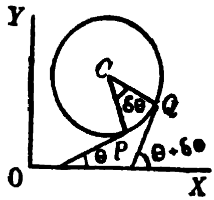


Fig. III-3

$$\sigma=\frac{\delta\theta}{\delta S}=\frac{\mathrm{PQ}/R}{\mathrm{PQ}}=\frac{1}{R}$$

Hence the curvature of a circle is equal to the reciprocal of the radius of circle.

**Curvature of a curve.** To find the curvature at a point on a curve, we describe a circle touching the point. Then the reciprocal of the radius of this circle (which we shall call the circle of curvature) is equal to the curvature of the curve at the point under consideration.

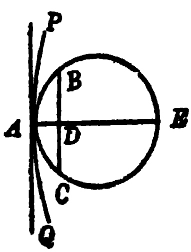


Fig. III-4

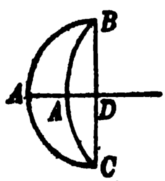


Fig. III-5

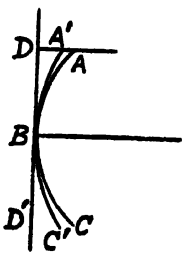


Fig. III-6

**The sagitta.** In (Fig. III-4) let ABECA be the circle of curvature at the point A of the curve PAQ. Then by the

well-known geometrical property of the circle,

$$BD^2=AD.DE=AD(2R-AD)$$

If AD is very small in comparison with $R$, the radius of the circle, then

$$BD^2=2R.AD,$$

or,

$$AD=\frac{BD^2}{2R}.$$

This length AD is called the *sagitta* of the circle.

If two circles of curvature BAC and BA′C touching two curves at the points A and A′ (as in Fig. III-5) have a common chord, and if the radii of these circles are $R$ and $R'$ respectively, then the sagitta

$$AD=\frac{BD^2}{2R}$$

and the sagitta

$$A'D=\frac{BD^2}{2R'},$$

Hence

$$\frac{AD}{A'D}=\frac{R'}{R}=\frac{\sigma}{\sigma'},$$

where $\sigma$ and $\sigma'$ are the curvature at the points A and A.′

If two curves touch at a common point B, so that, they have a common tangent BD (as in fig. III-6), then the two circles of curvature ABC and A′BC′ at the point B will have the same common tangent BD. By the property of circle,

$$BD^2=AD.\ 2R$$

where $R$ is the radius of the circle of curvature ABC. Hence the sagitta

$$AD=\frac{BD^2}{2R}$$

Similarly, for the circle of curvature A′BC′ with radius $R'$, the sagitta

$$A'D=\frac{BD^2}{2R'}$$

Hance

$$\frac{AD}{A'D}=\frac{R'}{R}=\frac{\sigma}{\sigma'},$$

$\sigma$ and $\sigma'$ being the curvatures of the circles ABC and A′BC′.

**Reflection of plane waves at a spherical surface.** In Fig. III-7 BAC is the section of the convex reflecting surface by the plane of the paper; ODA is the principal axis of the surface with centre at O. Let XDY be the section of the plane wave-front moving in the direction of the arrowheads. As the incident plane wave moves towards the mirror from left to right, it first touches the mirror at its pole A which thus becomes the source of secondary wavelets. As the wave moves further on, different points of the reflecting surface on either side of A become successive centres of secondary disturbance. When the plane wave occupies the extreme position XBCY, the extreme points B and C of the surface are just on the point of being disturbed. By Huygens' principle the envelope BA'C of all the secondary wavelets is the new reflected wave-front when the plane incident wave-front occupies the position XDY which it would occupy, if the incident wave-front were unobstructed by the reflecting surface. The reflected wave-front is therefore, spherical and it appears to diverge from its centre F, which is, therefore, the virtual image of the distant object.

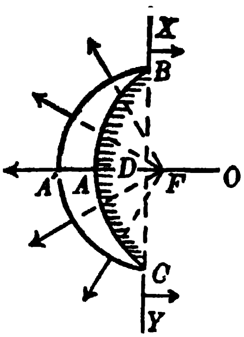


Fig. III-7

Since the reflected wave-front moves in the same medium as the incident wave-front,

$$A'A = AD$$

$$A'D = 2.AD$$

Since AD and A'D are the sagitta of the spherical surfaces BAC and BA'C which are supposed to be very small, we have

$$\frac{R'}{R} = \frac{AD}{A'D} = \frac{1}{2}$$

where $R$ and $R'$ are the radii of curvature of the surfaces BAC and BA'C. Thus, the centre F of the reflected wave-front is half way between the centre O of the reflecting surface and its pole A. Thus, the focal length of the convex mirror is equal to half the radius of curvature of the mirror.

**Concave spherical surface.** In the case of reflection of plane wave XDBY at a concave spherical surface A'BC' (Fig.

III-8) with centre O, the envelope of the secondary wavelets is part of a sphere ABC converging to the centre F (See Fig. 1, Plate II). Since the incident and the reflected waves move in the same medium, and, therefore, with the same velocity,

$$A'A = A'D$$

or $$AD = 2.A'D$$

so that $$\frac{A'D}{AD} = \frac{1}{2} = \frac{R}{R'},$$

*R* and *R'* being the radii of curvature of the surfaces ABC and A'BC'. So, in this case too, the focal length is equal to half the radius of curvature of the reflecting surface; or, calling $R = f$, the focal length of the mirror is

$$f = \frac{R'}{2}.$$

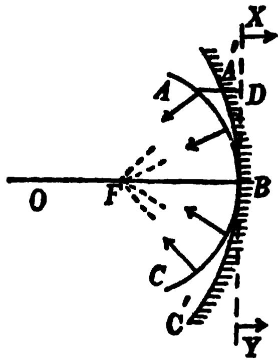


Fig. III-8

**Reflection of spherical waves at a spherical surface.** (*a*) *Reflecting surface convex.* In Fig. III-9, O is the centre of the convex reflecting surface BA'C of radius *R;* P is the centre of the incident spherical wave BA″C. The incident wave moving from left to right first touches the pole A' of the reflecting surface. As the wave moves onwards, points on either side of A' become successive centres of disturbance which move with the same velocity as the incident wave. When the incident wave occupies the position BA″C shown in the figure, the points B and C of the reflecting surface are just on the point of being disturbed. By Huygens' principle, the envelope BAC is the position of the reflected wave-front at this instant. This reflected wave front appears to diverge from the point *p*, which is, therefore, the image of the point P.

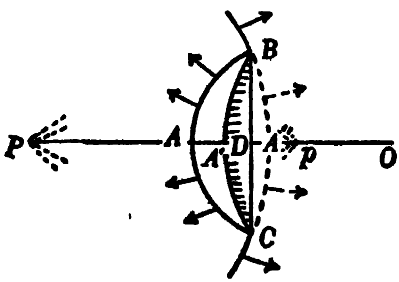


Fig. III-9

Now, from the geometry of the figure, we have

$$AA'=A'A''$$

or, $$DA-DA'=DA'+DA''$$

or, $$DA-DA''=2.DA'.$$

Since all the arcs have the same chord BDC,

$$\frac{BD^2}{2R}-\frac{BD^2}{2R''}=2.\frac{BD^2}{2R'} \quad .. \quad (16{\cdot}2)$$

where $R$, $R'$, and $R''$ are the radii of curvature of the surfaces BAC, BA'C, and BA"C respectively. From Eqn. (16·2)

$$\frac{1}{R}-\frac{1}{R''}=\frac{2}{R'}$$

Since the aperture of the mirror is supposed to be very small points A, A' and A" are very close together, so that, PA" is, approximately equal to PA', and $p$ A is approximately equal to A' $p$. Hence $R$ and $R''$ are approximately the object and the image-distances measured from the pole A' of the mirror. By the notation of geometrical optics

$$\frac{1}{v}-\frac{1}{u}=\frac{2}{R'}=\frac{1}{f}.$$

(*b*) **Reflecting surface concave.** This case is represented in Fig. III-10. Here BA'C is the concave reflecting surface with centre O. The incident wave-front diverges from the point P. After reflection, it remains spherical but converges to the point $p$. Since all the spherical surfaces BAC, BA'C and BA"C have a common chord BDC, and since the reflected and the incident waves move in the same medium, we have

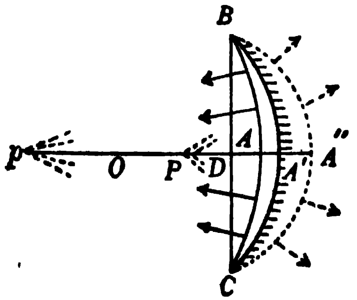


Fig. III-10

$$AA'=A'A''$$

or, $$DA'-DA=DA''-DA'$$

or, $$DA''+DA=2DA'$$

If $R$, $R'$, and $R''$ are the radii of curvature of the surfaces BAC, BA'C, and BA"C respectively, we have

$$\frac{1}{R''}+\frac{1}{R}=\frac{2}{R'}$$

Since the aperture of the mirror is supposed to be small, PA″=PA′=$u$=the object-distance and $p$A′=$p$A=$v$=the image-distance. Hence

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{R'}=\frac{1}{f}.$$

**Refraction of spherical waves at a plane surface.** Let BDC (Fig. III-11) be the plane surface separating two media (1) and (2), the medium (2) being optically denser than the medium (1). Let BA′C be the incident spherical wave diverging from the point P. Since the velocity $C_2$ of the wave in medium (2) is less than the velocity $C_1$ in medium (1), the envelope of the secondary wavelets would occupy the position BAC at the instant when the incident wave would have occupied the position BA′C, if it were moving in the medium (1). Hence, by the time $t$ the incident wave would move over the distance DA′ $(=C_1t)$ in the medium (1), the secondary disturbance originated at D would move over the distance DA $(=C_2t)$ in the medium (2), so that

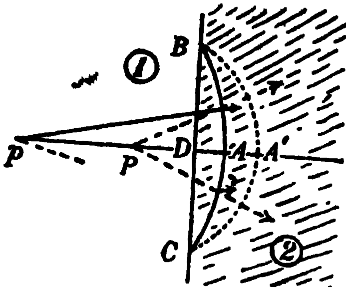


Fig. III-11

$$\frac{DA'}{DA}=\frac{C_1}{C_2}=n_{12}.$$

Since the surface BAC and BA′C have a common chord BDC

$$\frac{DA'}{DA}=\frac{R}{R'}$$

where $R$ and $R'$ are the radii of curved surfaces BAC and BA′C respectively. Hence

$$R=n_{12}R'$$

Since the aperture of the refracting surface is supposed to be small, PD=PA′=$R'$, and $p$A=$p$D =$R$ (approximately) so that

$$pD=n_{12}.PD$$

**Refraction of plane waves at a spherical surface.** (*a*) *Refracting surface convex.* Ih Fig. III-12 BA′C represents

the convex spherical surface separating two media (1) and (2), and having its centre at O. Let the medium (2) be optically denser than the medium (1). Hence the incident plane wave XBDCY, after refraction is transformed into spherical wave BAC with centre at F which, is the principal focus. Let $C_1$ and $C_2$ be the velocities of propagation of disturbances in the media (1) and (2) respectively, so that

$$A'D = C_1 t, \text{ and } A'A = C_2 t.$$

or, $$\frac{A'D}{A'A} = \frac{C_1}{C_2} = n_{12}.$$

But $$AA' = DA' - DA$$

Hence, $$DA' = n_{12}(DA' - DA)$$

or, $$n_{12}.DA = (n_{12} - 1)DA'$$

or. $$\frac{DA}{DA'} = \frac{n_{12} - 1}{n_{12}} = \frac{R'}{R} = \frac{OA'}{AF}$$

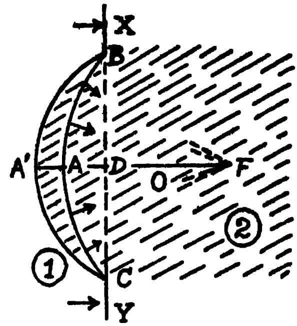


Fig. III-12

where $R$ and $R'$ are the radii of the surfaces BAC and BA'C. Since the aperture of therefracting surface is supposed to be small, FA=FA' (approx.).

Hence, $$\frac{OA'}{FA'} = \frac{n_{12} - 1}{n_{12}}$$

or, calling FA', the focal length $f$ of the refracting surface

$$\frac{1}{f} = \frac{n_{12} - 1}{n_{12}.R'}$$

where $R'$ is the radius of curvature of the refracting surface.

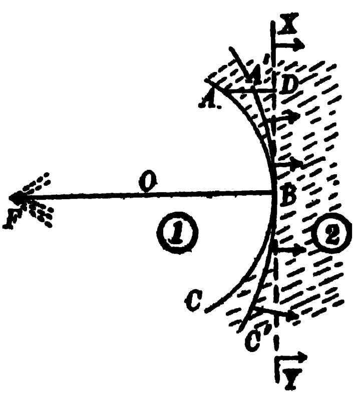


Fig. III-13

(*b*) *Refracting Surface concave.* In Fig. III-13, ABC is the concave refracting surface separating two media (1) and (2), the medium (2) being denser than medium (1). XDBY shows the position the incident wave would have occupied, if it were not obstructed by the medium (2), while A'BC' is the position of the refracted wave-front at the

same instant. In this case

$$\frac{\mathrm{AD}}{\mathrm{AA'}}=\frac{C_1}{C_2}=n_{12}$$

where $C_1$ and $C_2$ are the velocities of the disturbances in the media (1) and (2) respectively.

| | |
|---|---|
| But | $\mathrm{AA'}=\mathrm{DA}-\mathrm{DA'}$ |
| so that | $\mathrm{AD}=n_{12}(\mathrm{DA}-\mathrm{DA'})$ |
| Hence, | $n_{12}\mathrm{DA'}=(n_{12}-1)\mathrm{DA}$ |
| or, | $\dfrac{\mathrm{DA'}}{\mathrm{DA}}=\dfrac{R}{R'}=\dfrac{n_{12}-1}{n_{12}}$ |

where $R$ and $R'$ are the radii of curvature of the surfaces ABC and A'BC' respectively. The centres of curvature of the surfaces ABC and A'BC' being Q and F respectively,

$$\frac{R}{R'}=\frac{\mathrm{OB}}{\mathrm{BF}}=\frac{n_{12}-1}{n_{12}}$$

or,

$$\frac{1}{f}=\frac{n_{12}-1}{n_{12}R}$$

where $f$ is the focal length and $R$ is the radius of curvature of the refracting surface.

**Refraction of spherical waves at a spherical surface.** Let BAC (Fig. III-14) be a concave spherical surface separating two media (1) and (2), the medium (2) being denser than the medium (1). Let O be the centre of the refracting surface. In this figure BA″C gives the position of the unobstructed incident wave-front with centre P at the instant $t$ and BA′C is the position of the refracted wave-front with centre $p$ at the same instant. If $C_1$ and $C_2$ be the velocities of propagation of waves in the media (1) and (2),

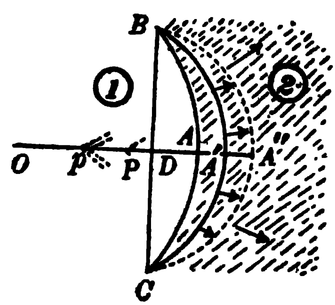


Fig. III-14

$$\frac{\mathrm{AA''}}{\mathrm{AA'}}=\frac{C_1}{C_2}=n_{12}$$

| | |
|---|---|
| But, | $\mathrm{AA''}=\mathrm{DA''}-\mathrm{DA}$ and $\mathrm{AA'}=\mathrm{DA'}-\mathrm{DA}$ |
| Hence, | $(\mathrm{DA''}-\mathrm{DA})=n_{12}(\mathrm{DA'}-\mathrm{DA})$. |

Since all the arcs have a common chord BDC

$$\frac{BD^2}{R''} - \frac{n_{12}.BD^2}{R'} = (1-n_{12})\frac{BD^2}{R}$$

or,
$$\frac{1}{R''} - \frac{n_{12}}{R'} = \frac{1-n_{12}}{R} \quad .. \quad (16\cdot4)$$

where $R$, $R'$ and $R''$ are the radii of curvature of the surfaces BAC, BA′C and BA″C respectively. Since the aperture of the refracting surface is supposed to be small PA″=PA=$u$, the object-distance and $p$A′ =$p$A=$v$, the image-distance (approximately). Hence from Eqn. (16.4)

$$\frac{1-n_{12}}{u-v} = \frac{1-n_{12}}{R}. \quad .. \quad (16\cdot5)$$

**Refraction through a lens.** In establishing the relation between conjugate distances in the case of a lens with the help of the Fermat's principle in Geometrical Optics (see Part 1, P. 75) we assumed that the equivalent paths PAQ and PMNQ in Fig. III-15 are equal. This fact follows easily from the following consideration.

Let RML be the wave-front with centre at P and incident on the lens at the point M. Since the medium of the lens is supposed to be optically denser than the surrounding medium, a disturbance moves with a slower speed through the lens than through the surrounding medium. Hence the envelope SNT on emergence becomes curved in the opposite direction (See Figs. 2 and 4 Plate III) with centre at Q, which is, therefore the image of P. Since phase has got the same value at every point on

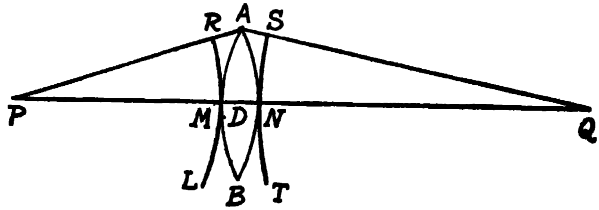


Fig. III-15

a wave-front, the difference of phase between two corresponding points in the two wave-fronts RML and SNT must be the same

for all pairs of corresponding points. In other words, time taken by a disturbance to move from R to S along the path RAS must be the same as that required for moving from M to N along the path MDN through the lens. Hence, if $C_1$ and $C_2$ be the velocities of disturbances through the surrounding medium and through the lens

$$\frac{\text{Path (RAS)}}{C_1}=\frac{\text{Path (MDN)}}{C_2}$$

or, $$\text{Path (RAS)}=\frac{C_1}{C_2}.\ \text{Path (MDN)}$$

or, $$\text{(RAS)}=n_{12}.\ \text{(MND)}$$

Again since RML and SNT are portions of spheres with centres at P and Q respectively, we have

$$PR=PM,\ \text{and}\ QS=QN$$

or, $$PR+QS=PM+QN$$

or, $$PR+RAS+SQ=PM+QN+n_{12}.MN$$

or, $$PA+AQ=PM+MN+QN+(n_{12}-1)MN$$

or, $$PA+AQ=PMNQ+(n_{12}-1)MN.$$

The relation between conjugate distances follows easily as in Geometrical Optics.

# CHAPTER IV

## INTERFERENCE OF LIGHT

**17. Principle of Superposition.** Hitherto we had been chiefly concerned with changes introduced in the shape of the wave-fronts after passing through the surface of separation between two homogeneous media. Assuming Huygens' view of the mechanism of wave propagation, we have deduced almost all the laws of geometrical optics. In this chapter we propose to study the behaviour of waves when two trains of waves starting from two point sources move over the same part of the medium, thus criss-crossing one another on their way. At any instant a particle of the medium traversed by the two trains of waves would have two displacements, since it would lie on the two wave-fronts simultaneously, and the particle under consideration would necessarily move in the direction of the resultant displacement according to the principle of superposition which runs as follows :

**Principle of superposition.** *When a particle of a medium is struck simultaneously by any number of waves, its displacement at any instant is given by the algebraic sum of displacements that would be produced by each acting separately.*

In physical optics, by interference of light is meant the effects of superposition of waves generated by two point or linear sources which must be coherent, *that is, stand to each other in definite phase relation for all times.*

**Analogy of ripples.** The idea of interference of waves is best illustrated by considering the case of ripples on water surface. In Fig. 6, Plate III, two trains of ripples are shown to be produced by two sources vibrating simultaneously in the same phase and with the same amplitude. It will be noticed that over certain lines which are regularly spaced on the surface of water

the amplitudes have been added up. In the intervening spaces the two vibrations have neutralised their effects over similarly spaced lines, so that, the displacements over these lines is zero. It will also be noticed that these lines of maximum and minimum displacements are members of a family of hyperbolas having the two sources as their foci.

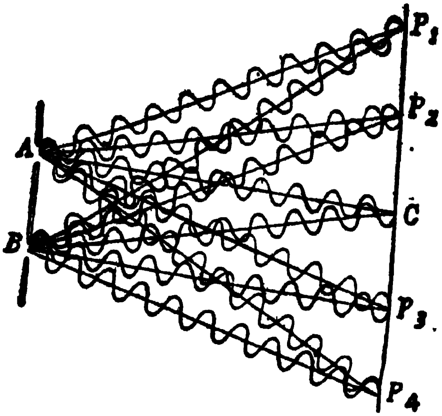



Fig. IV-1

Since light energy is carried from one place to another by means of waves through the medium of ether, we may naturally expect that two trains of light waves produced by two sources may neutralise the effect of each other so as to produce darkness (Fig. IV-1) which corresponds to zero displacement in the case of interfering ripples mentioned above. We have seen in the

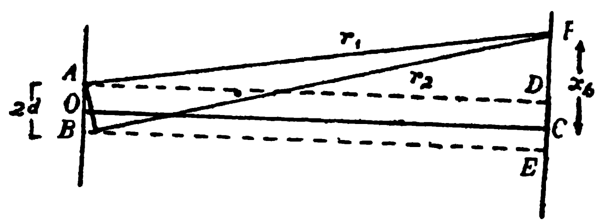



Fig. IV-2

introductory chapter how these expectations were realised by Young, Fresnel, and others, thus providing additional support to the view that light energy is carried by ether waves.

**Interference of Waves from Two Coherent Sources.** Referring to Fig. IV-2, let the distance between the sources A and B be $2d$ and let the distance OC of the line AB from the screen be $c$, O being the mid-point of AB. Let also the distance CP be $x_s$.

Now from Fig. IV-2,

$$r_1^2 = c^2 + (x_s - d)^2$$

$$r_2^2 = c^2 + (x_s + d)^2$$

$$r_2^2 - r_1^2 = (x_s + d)^2 - (x_s - d)^2 = 4x_s d$$

or, $$r_2 - r_1 = \frac{4x_s d}{r_1 + r_2}.$$

The length $x_s$ is very small compared to $r_1$ and $r_2$, and the point P is very close to C, so that, to a very close approximation $r_1 + r_2 = 2c$. Thus

$$r_2 - r_1 = \frac{2x_s d}{c}. \qquad \ldots \quad (17 \cdot 1)$$

In order that the point P may be a point of maximum brightness, the two waves must meet at P in the same phase, or the path difference must be an even number of half wave-lengths, (See Fig. IV-I)

or, $$r_2 - r_1 = \frac{2x_s d}{c} = 2s.\frac{\lambda}{2}$$

or, $$(x_s)_{bright} = \frac{cs\lambda}{2d} \quad [s = 0, 1, 2, 3, 4, \ldots] \qquad (17 \cdot 2)$$

This equation (17·2) gives the distance of $s^{th}$ bright band from the centre C. Similarly for the $s^{th}$ dark band, the two waves must meet in opposite phases, or the path-difference must be an odd number of half wave-lengths.

or, $$r_2 - r_1 = \frac{2x_s d}{c} = (2s+1)\frac{\lambda}{2}$$

or, $$(x_s)_{dark} = \frac{(2s+1)\lambda c}{4d} [s = 0, 1, 2, 3, 4, \ldots] \; (17 \cdot 3)$$

5D

**The centre of the fringe.** At the point C, where $r_1=r_2$. all waves meet in the same phase, and therefore, the intensity is the maximum. If the interfering beam be of white light, the central fringe would be necessarily white.

**18. Characteristics of Interference Fringes. Interference fringes are equi-spaced.** Taking the case of bright fringes, the distance between the $s$th and the $(s+1)$ *th* bright fringe is

$$x_{s+1}-x_s=\frac{c(s+1)\lambda}{2d}-\frac{cs\lambda}{2d}$$

or, $$x_{s+1}-x_s=\frac{c\lambda}{2d}=\text{constant} \quad .. \quad (18\cdot1)$$

The width of a fringe can, therefore, be increased by increasing the distance $c$ between the sources and the screen and also by diminishing the distance between the interfering sources.

The right-hand side of this equation (Eqn. 18 1) is constant and independent of $s$, the order of the fringe. Hence, the distance between consecutive bright fringes is the same over the entire length of the fringe system. Similarly, the distance between two consecutive dark fringes is

$$x_{s+1}-x_s=\frac{c(2s+3)\lambda}{4d}-\frac{c(2s+1)\lambda}{4d}=\frac{c\lambda}{2d} \quad .. \quad (18\cdot2)$$

which is the same as Eqn. (18·1). Hence, the interference fringes are alternately bright and dark and they are equi-spaced.

**Dependence on wave-length.** For a given value of $c$ and $d$, the width of the bright band would depend on the wave-length $\lambda$ (Eqn. 18·1). Since the wave-length for red light is nearly double that for violet light, width of red light band will be nearly double that for violet light. If the interfering light is composed of two wave-

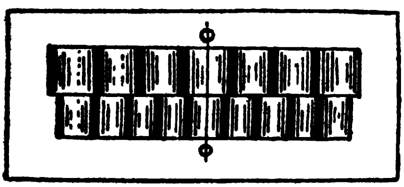


Fig. IV-3

lengths $\lambda_1$ and $\lambda_2$, the fringes corresponding to these wave-lengths, if they could be separated out, would present an appearance like that shown in Fig. IV-3. The intensity curve for these two wave-lengths would be as represented by the two curves shown in Fig. IV-4. There is no point in the resultant intensity curve where the intensity is zero; it only fluctuates between maximum and minimum values. This variation gradually diminishes as the distance from the centre of the band increases. Evidently, if the interfering beams be each composed of a large number of wave lengths as in the case of white light, the regions of maximum intensity of the same order for all wave-lengths would not fall on the same line. Hence, in the neighbourhood of the centre of the fringe each band will spread out into spectral colours. But as the distance from the centre increases,


Fig. IV-4

the overlapping of colours increases, the result being a general illuminations. Under this circumstance only a few coloured fringes will be observed.

The centre C of the fringe system with white light must be white. Since, at this point rays of all colours meet in the same phase, there is perfect overlapping of all colours at the centre, and the colour of the central fringe is necessarily white.

**Spectroscopic eamination of interference fringes.** If the narrow slit of a spectroscope is placed on the central band of the interference fringe system formed with white light, we get the complete continuous spectrum in the field of view of the telescope. As the slit is moved slowly away from the central band, at first a dark line appears at the violet end of the spectrum which moves across the field from

violet towards the red. At this point the violet reappears in maximum intensity. Beyond this point the dark bands become more and more numerous and finally merge into one another and the field of the telescope becomes dark.

**Method of finding the centre of a fringe system.** The above consideration leads to a very simple method of locating the centre of a fringe system formed with monochromatic light. With such light, the whole field is covered with uniformly coloured similar fringes and it is not possible to locate the central fringe. To find the position of the central fringe all that we have to do is to replace the two interfering sources by two sources of white light, when a white fringe will appear at the centre. A more convenient method of locating the central fringe is to illuminate the two slits simultaneously with white light of lesser intensity placed behind the two monochromatic sources. Under this circumstance the central fringe will come out as a white band superposed on the uniformly coloured fringe system formed with monochromatic light.

**Conditions for obtaining large number of fringes.** It follows that to obtain large number of fringes, the light must be monochromatic. Red line of cadmium having wave-length $6438{\cdot}4696 \times 10^{-8}$cm. is found to be strictly monochromatic, that is, emitting radiation of one particular wave-length. For all practical purposes sodium light may be used; but it consists of two wave-lengths $5890 \times 10^{-8}$ cm. and $5896 \times 10^{-8}$ cm. known as $D_2$ and $D_1$ lines which are very close together. The other necessary condition is that the intensity of the interfering sources must be high.

**Energy consideration.** It should be noted that by 'intensity being zero at the minimum points' we do not mean that at these points light energy is annihilated for, this would be a violation of the principle of energy. The fact is that in this case, the distribution of energy is altered, the energy being accumulated at one place at the expense of energy at other places.

**19. Conditions for Interference.** Let the equations of vibration of the two interfering sources at the point P on the screen (Fig. IV-2) be

$$y_1 = a \sin 2\pi \left\{ \frac{t}{T} - \frac{r_1}{\lambda} + \alpha_1 \right\} \quad \ldots \quad (19\cdot1)$$

and

$$y_2 = b \sin 2\pi \left\{ \frac{t}{T} - \frac{r_2}{\lambda} + \alpha_2 \right\} \quad \ldots \quad (19\cdot2)$$

where $a$ and $b$ are the amplitudes of vibration at the point P, $r_1$ and $r_2$ are the distances of the point P from the two sources. Let $\alpha_1$ and $\alpha_2$ be the phases at the two sources at the instant $t=0$. Let $\alpha_1$ and $\alpha_2$ be independent of one another. If the two sources of light be two candle flames, for example, then the light coming from a particular point of a flame is due to a number of radiating particles occupying the point in succession. These particles have no definite phase relation between them. Accordingly, the phase of vibration at the point P under consideration may change abruptly and irregularly many times per second.

The resultant displacement due to the two interfering point sources at the point P obtained by superposition is

$$y = y_1 + y_2$$

or, $$y = a \sin 2\pi \left\{ \frac{t}{T} - \frac{r_1}{\lambda} + \alpha_1 \right\} + b \sin 2\pi \left\{ \frac{t}{T} - \frac{r_2}{\lambda} + \alpha_2 \right\}$$

Putting $\theta = \frac{2\pi t}{T}$, $\beta_1 = 2\pi \frac{r_1}{\lambda} + \alpha_1$ and $\beta_2 = 2\pi \frac{r_2}{\lambda} + \alpha_2$

$$y = a\,(\sin\theta\cos\beta_1 - \cos\theta\sin\beta_1) + b\,(\sin\theta\cos\beta_2 - \cos\theta\sin\beta_2)$$

or, $y = \sin\theta\,(a\cos\beta_1 + b\cos\beta_2) - \cos\theta\,(a\sin\beta_1 + b\sin\beta_2)$

Put $$a\cos\beta_1 + b\cos\beta_2 = A\cos\delta$$
and $$a\sin\beta_1 + b\sin\beta_2 = A\sin\delta$$

Then $$y = A\,(\sin\theta\cos\delta - \cos\theta\sin\delta)$$

or, $$y = A\sin(\theta - \delta) = A\sin\left(\frac{2\pi t}{T} - \delta\right)$$

The resultant displacement is, therefore, simple harmonic. The amplitude of the resultant vibration is given by

$$A^2=(a\cos\beta_1+b\cos\beta_2)^2+(a\sin\beta_1+b\sin\beta_2)^2$$

or $$A^2=a^2+b^2+2ab\cos(\beta_1-\beta_2)$$

or $$A^2=a^2+b^2+2ab\cos 2\pi\left\{\frac{r_1-r_2}{\lambda}-(\alpha_1-\alpha_2)\right\} \qquad \ldots \quad (19{\cdot}3)$$

The epoch of the resultant displacement is given by

$$\tan\delta=\frac{a\sin\beta_1+b\sin\beta_2}{a\cos\beta_1+b\cos\beta_2} \qquad \ldots \quad (19{\cdot}4)$$

Equations (19·3) and (19·4) give all the conditions necessary for interference producing a large number of well defined interference fringes

The period $T$ and the wave length $\lambda$ in the equations for interfering waves is taken to be the same for both the waves. If the wave-lengths of the interfering sources be different, the intensity at a point on the screen would vary with time (compare, for example, the phenomenon of beats) and the average effect at the point would be general illumination. Hence the first condition for interference is

**condition 1.** *The two interfering sources must be exactly similar in respect of frequncy and wave-length.*

The intensity of light at a point is proportional to the square of the amplitude of vibration at the point, that is proportional to $A^2$ (see Eqn. 13·3). Evidently, the maximum and minimum values of intensity at the point due to the two interfering sources are $(a+b)^2$ and $(a-b)^2$. For *destructive interference* (zero minimum intensity) $a$ must be equal to $b$. Under this condition the maximum value of intensity is $4a^2$ that is, four times the intensity due to a single source.

Now, intensity of light at a point falls off inversely as the square of the distance of the point from the source. Starting

with two sources of equal intensity, the interference is destructive only in the neighbourhood of the central fringe. At greater distances on the screen from the centre the condition of destructive interference ($a=b$) is not satisfied and the fringes become more and more indistinct till they produce uniform illumination. With two sources of equal intensity placed close together the fringes remain distinct over a considerable range and the width of the fringes increases. Hence the second condition of interference is

**condition 2.** *To obtain a large number of distinct fringes, the the two sources of interference must be of equal intensity and they must be placed very close together.*

For maximum intensity

$$\cos 2\pi\left\{\frac{r_2-r_1}{\lambda}-(\alpha_1-\alpha_2)\right\}=1$$

This requires $2\pi\left\{\frac{r_2-r_1}{\lambda}-(\alpha_1-\alpha_2)\right\}=2s\pi$

where $s=0, 1, 2, 3$, etc.

At a given point on the screen ($r_1$ and $r_2$ remain constant) the intensity will be determined by $(\alpha_1-\alpha_2)$. If the phases $\alpha_1$ and $\alpha_2$ of the interfering sources change abruptly and irregularly, the intensity at the point under consideration can never remain constant. This is true for all points on the screen where the two interfering beams overlap. Irregular changes of phase would necessarily cause irregular variation of intensity many times per second whose average effect on the eye on account of the persistence of visual impression is general illumination. Hence follows the third condition of interference :

**condition 3.** *To obtain permanent and steady interference fringes, the two interfering sources must be maintained at constant difference of phase for all times.*

Interfering sources which are maintained at a constant difference of phase are said to be **coherent.** Sources whose phases change irregularly are called **incoherent** sources. It can be proved that the illumination produced at a point by a number of incoherent sources is equal to the sum of the illumination produced by the sources acting separately at the point.

**20. Methods of Producing Coherent Sources.** Two coherent sources capable of interference can be obtained in either of the two ways described below :

(1) **By division of wave-front.** Since a wave-front is the locus of points on a surface which are in the same phase of vibration at an instant, any two portions of the same wave-front are always in the same phase relatively to one another. Hence they are in a position to produce interference fringes. This method of division of wave-front was used by Young for the production of interference fringes. He admitted a thin pencil of light through a narrow slit in a shutter. The pencil fell upon a screen perforated with two narrow parallel slits placed very close together, thereby allowing two narrow portions of the same wave-front to be isolated and propagated through the slits. These transmitted wave-fronts produced interference fringes on a screen in the region where the pencils overlapped.

(2) **By reflection or refraction.** If the two interfering sources stand to one another in the relation of object and its image or if two images of the same source are formed by reflection or refraction, then between two such sources there would exist a permanent phase relation and they would, therefore, be in a position to interfere. This method is generally used for the production of interference sources in the laboratory.

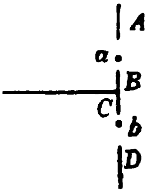


Fig. IV—5

To take a specific example, let us suppose one of the sources is a slit AB (Fig IV-5) perpendicular to the plane of the paper and illuminated by monochromatic light; also let CD be its image of the same size formed by reflection or refraction, the two sources being parallel and placed close together. In this case the interfering sources will consist of a multitude

of pairs of conjugate points lying in AB and CD. Each pair of such points would produce a set of interference fringes. Two cases will have to be considered here.

**Case 1.** The object and its image are laterally inverted as by reflection at a plane mirror. In this case any two conjugate interfering point sources such as *a* and *b* are equidistant from the line of symmetry (Fig. IV-5), so that, the position of the central fringe would be the same for each


Fig. IV—6

pair of interfering points, but the distance ($2d$) between them would be different. Since the width of a band diminishes as $d$ increases (Eqn. 18·2) there would be no point of zero intensity and the fringes would be indistinct and the confusion would increase from the centre outwards unless the slit is very narrow. This will be clear from the study of the intensity curves in Fig. IV-6 where O represents the centre of the fringe systems.

**Case 2.** When the pairs of conjugate points such as *a* and *b* (as in Fig. IV-7) are equidistant from the edges A and C, the distance between conjugate pairs of points is the same for all pairs but their centres of symmetry are different for different pairs. In Fig. IV-8, curves I and II show the intensity curves for pair of points A, C and B, D; curves for other pairs of conjugate points are placed in intermediate positions. It follows that the width of band increases as the slit width AB increases. Hence if AB is

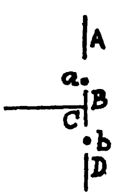



Fig. IV—7

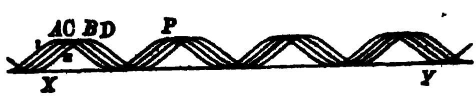



Fig. IV—8

equal to AP, the distance between consecutive maxima for any conjugate pair of points, all the separate bands would coalesce

and the result would be general illumination. Under this condition the width of the interfering sources cannot be increased beyond a certain limit.

We now proceed to describe some of the specific methods of producing interference fringes based on the second method of production of coherent sources described above.

**Lloyd's single mirror method.** The apparatus consists of optically worked plane mirror of metal or of black glass, so that, there can be no internal reflection. The source of light is a fine slit illuminated by sodium light and it is placed parallel to the plane of the mirror (Fig. IV-9). The slit being placed close (one to two millimetres from one edge of the mirror) to the reflecting surface, the light falls on the mirror almost at grazing incidence.

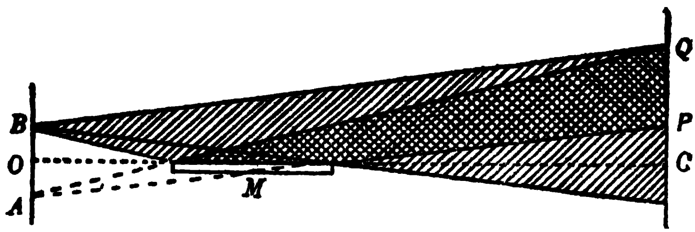



Fig. IV—9

In this case the two interfering sources are (i) the illuminated slit B and (ii) its virtual image A formed by reflection at the mirror. Over the portion PQ of the screen, the two beams—one coming directly from the slit B and the other coming from the virtual image A of the slit overlap. These two overlapping beams produce interference fringes on a screen placed perpendicularly to the plane of the mirror, the fringes being parallel to the length of the slit.

**Characteristics of Lloyd-mirror fringes.** (1) In this arrangement the line OMC is the line of symmetry, the point C being equidistant from the two interfering sources. Since there is no overlapping of direct and reflected beams at C, the central

inge does not appear here on the screen. The fringe system begins to appear at P and extends right up to Q. Hence with Lloyd's single mirror only less than half of the complete system of fringes can be obtained.

(2) Dr. Lloyd found that the whole system of bands are displaced by the width of half a band from the position which they would occupy according to the simple theory given in Art 17. This is due to change in phase by 180° on account of the change in condition of reflection (that is, from denser to a rarer medium, See Art. 23). For this reason the central fringe would be dark.

(3) Since one of the interfering beams is obtained by reflection. at a plane mirror, the virtual image is laterally inverted with respect to the slit. The line of symmetry for every pair of interfering point sources in the object and the image are the same. If the slit width is small, the width of the bands due to each pair of corresponding point sources would be nearly the same, so that, all the maxima of the same order would be superposed together and so also the minima of the same order. Hence the fringes are fairly sharp and a fairly wide slit can be used.

**Fresnel's ouble-Mirror Fringes.** The instrument consists of two plane parallel mirrors DM and DN inclined to one an other at an angle slightly less than 180°, so that they lie almost in the same plane. The source S (Fig. IV-10) consists of a narrow slit illuminated by sodium light and placed parallel to the line of intersection of the two mirrors. The light reflected from the two mirrors overlap in the region PQ on a screen where the interference fringes are formed.

Light reflected from the two mirrors appear after reflection to diverge from two virtual sources A and B. In this case the source and the two images A and B lie on the circumference of a circle of radius DS. Let $\omega$ be the acute angle between the two mirrors. Join AS and BS and produce ND and DM to cut the

lines BS and AS at the points $m$ and $n$. Then BS and AS are the normals to the mirrors DN and DM respectively, so that, $\angle ASB = \omega$ and $\angle ADB = 2\omega$. The line OD bisecting the angle ADB is therefore, the line of symmetry and the point C on the screen

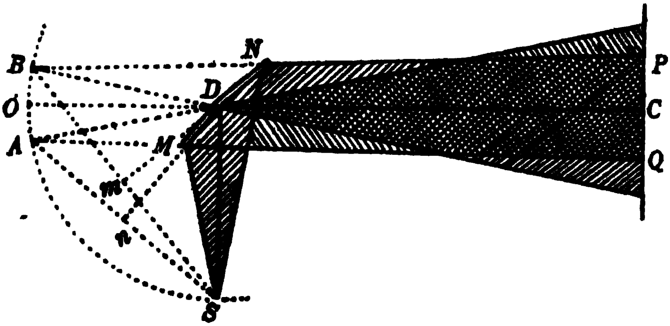



Fig IV-10

on the line OD produced is the centre of the fringe system. Denoting the distance OD and DC by $c$ and $b$ respectively, $AB = 2c \sin \omega$. Hence the distance of the $s^{th}$ bright fringe from the centre of the fringe system is by Eqn. (17·2)

$$x_s = \frac{(b+c)s\lambda}{2c \sin \omega}.$$

Since $\omega$ is very small, $\sin \omega = \omega$ and

$$(x_s)_{bright} = \frac{(b+c)s\lambda}{2c\omega} \quad [s=0,\ 1,\ 2,\ 3,\ 4 \ldots] \quad \ldots \quad (20{\cdot}1)$$

Similarly, the distance of the $s^{th}$ dark fringe from the centre is

$$(x_s)_{dark} = \frac{(b+c)\,(2s+1)\lambda}{4c\omega} \quad [s=0,\ 1,\ 2,\ 3,\ 4,\ldots]$$

**Characteristics of the bi-mirror fringes.** In this case the whole system of fringes on the two sides of the centre of symmetry can be observed.

Since both the interfering beams are reflected in the same manner (from glass to air), a change of phase equal to 180° occurs at reflection at each mirror (Art 23). Hence the position of the centre of the fringes coincides with the centre of symmetry.

Since the two virtual interfering images of the slit are similarly situated with respect to one another, the centres of symmetry of different pairs of corresponding interfering point sources are shifted relatively to one another. Hence the fringes are not very sharp.

**Fresnel's Biprism Fringes.** In this case two coherent sources are formed by refraction through two very thin prisms meeting over a common base, the two refracting edges of the prism being turned in opposite directions (Fig. IV-11)

Let a fine source of light (a narrow slit illuminated by sodium light, for example,) be placed in the plane passing through the common base of the prisms. Since the prisms are supposed to be very thin, they would produce two distinct images of the slit by refraction. In Fig. IV-12, FED is the section of the bi-prism by the plane of the paper and O is the source of light. The rays refracted through the prism FE appear after refraction to come from B. Similarly with respect to the prism DE, A is another virtual image of slit. Since the prisms are very thin, the two virtual interfering sources are very close to one another and are situated at the same distance from the prism as the source O. Rays diverging from the virtual sources A and B overlap in the region PQ on the screen where interference fringes are formed. The centre of symmetry of the fringes is situated at the point C lying on the line OE produced.

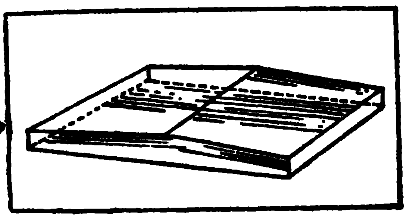


Fig. IV-11

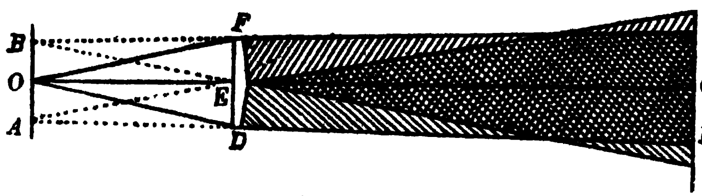



Fig. IV-12

The distance of the *s*th band from the centre of the system is deduced as follows : Let the distance OE and EC be *b* and *c*

respectively. Let $i$ be the refracting angle of the prism, then, the deviation produced by each prism is

$$\angle BEO = \delta = (n-1)i$$

where $n$ is the refractive index of material of the prism. Hence the distance OB is equal to

$$OB = b\delta = b(n-1)i$$

so that, the distance between the virtual sources is

$$AB = 2d = 2b(n-1)i$$

Substituting the values of OC and AB in Eqn. (17·2), the distance of the $s$th band from the centre of symmetry is

$$(x_s)_{bright} = \frac{(b+c)s\lambda}{2d} = \frac{(b+c)s\lambda}{2b(n-1)i} \quad [s=0, 1, 2, 3, 4 \ldots]$$

This shows that biprism fringes are similar to bi-mirror fringes, the angle of inclination of the mirrors corresponding to $\omega$ is $(n-1)i$

Similarly, the distance of the $s$th dark band from the centre is

$$(x_s)_{dark} = \frac{(b+c)(2s+1)\lambda}{4d} = \frac{(b+c)(2s+1)\lambda}{4b(n-1)i} \quad [s=0, 1, 2, 3, 4, \ldots]$$

**Characteristics of the bi-prism fringes.** (1) The bi-prism fringes are much brighter than those produced by reflection, since the prisms transmit almost the whole of the light incident upon them.

(2) Owing to dispersion in the glass, the distance between two virtual foci would be different for different colours. Thus, with white light the distance between the violet foci $A_v$ and $B_v$ would be greater than that between the red foci $A_r$ and $B_r$.

Denoting these distances by $2d_v$ and $2d_r$ respectively, the distance of the *s*th violet band from the centre is

$$x_{sv}=\frac{(b+c)s\lambda_v}{2d_v}$$

and that of the *s*th red band from the same point is

$$x_{sr}=\frac{(b+c)s\lambda_r}{2d_r},$$

$\lambda_v$ and $\lambda_r$ being the wave lengths of the violet and the red lights respectively. Hence

$$x_{sv}-x_{sr}=\frac{(b+c)s\lambda_v}{2\ \ d_v}-$$

Since $\lambda$ is nearly proportional to $d$, overlapping is much reduced and we get fairly large number of fringes with white light.

In the case of Fresnel's double mirror, there is, of course, no dispersion and the distance between *s*th violet and the *s*th red fringes is

$$x_{sv}-x_{sr}=\frac{(b+c)\lambda}{2c\omega}\left(\lambda_v-\lambda_r\right)$$

the distance $d=c\omega$ being the same for all colours. With a bi-mirror system the iris-coloured bands are broadened and overlapping is proportionately increased, so that, much lesser number of fringes are obtained with white light.

(3) As in the case of Fresnel's bi-mirror, the two virtual images formed by the two prisms are similarly situated with respect to one another. Hence the centres of symmetry for different pairs of conjugate point sources are different. Hence for distinctness the width of the slit must be narrow compared with the width of the band.

(4) The apearance of biprism fringes is complicated by the diffraction fringes which are superposed on the interference fringes. This will be discussed in its proper place.

**Billets' Split Lenses.** With Fresnel's biprism or bi-mirror, the two interfering sources are virtual images of the slit. In the split lens method, the interfering sources are two real images of a source produced by refraction through two half-lenses. In Fig. IV-13, L and L′ are two half-lenses slightly separated from one another with respect to the line of symmetry OC. An illuminated narrow slit is placed at O parallel to the bases of the lenses and two real images of the slit are formed at the points A and B which may be brought as close together as possible by moving the half-lenses L and L′ towards one another. Rays diverging from A and B placed symmetrically about the line of symmetry OM overlap in the region PQ of a screen where interference fringes are formed.

**Method of producing circular fringes.** By modifying Billet's experiment M. G. Mestin was able to produce circular

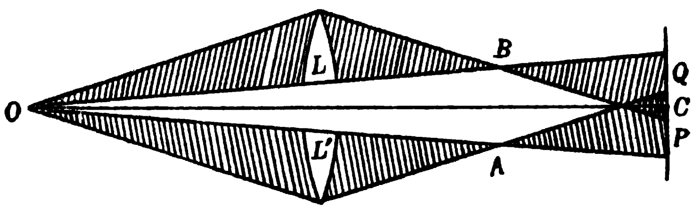



Fig. IV-13

fringes on a screen. In this experiment the positions of the split lenses were so adjusted, that the two real images of the source were formed on the line of symmetry and slightly displaced with respect to one another. The screen being placed perpendicularly to the line joining the two sources, the lines of equal intensity would intersect the screen in concentric circles. So the fringes are concentric circles round a common centre which lies on the line of symmetry OC. Since the interfering beams overlap only on one side of the line of symmetry, the fringes, however, are not complete circles and, at most, only half-circles can be obtained.

**21. Determination of Wave-length of Light by Fresnel's bi-prism method.** The distance between the $(s+n)^{th}$ fringe and the $s^{th}$ fringe produced by the Fresnel's biprism is

$$r = x_{s+n} - x_s = \frac{(b+c)(s+n)\lambda}{2d} - \frac{(b+c)s\lambda}{2d}$$

or,

$$r = \frac{(b+c)n\lambda}{2d}$$

or,

$$\lambda = \frac{2rd}{n(b+c)} \quad \cdots \quad \cdots \quad (21{\cdot}1)$$

where $2d$ is the distance between the two virtual interfering sources, $c$ is the distance between the source and the prism and $b$ is the distance between the prism and the screen or the measuring eye-piece. Hence the determination of $\lambda$ involves the determination of $r$, $d$, and $(b+c)$.

**The apparatus.** It is a special type of optical bench shown in Fig. IV-14. It consists of a heavy metal base standing on four levelling screws and provided with a scale graduated in millimetres all throughout its length. On this bed there are four sliding uprights C,D,E, and F each provided with a vernier moving over the scale of the bench. The upright C carries an adjustable slit S supported on two jaws by which the slit can be moved in a direction perpendicular to the length of the bench. The slit can also be rotated about a horizontal axis parallel to the length of the bench by means of a tangent screw T. The slit with its accessories is mounted on a frame which can be moved up or down by a rack and pinion arrangement.

The upright D is similar to C, only the slit is replaced here by the biprism P. In addition, the upright stand carrying the biprism can be moved at right angles to the length of

the bench by means of a screw $S_1$ attached to the base of the upright.

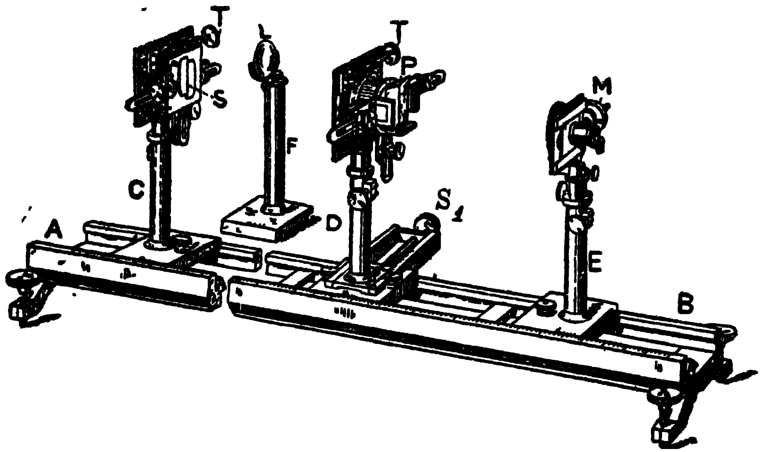



Fig. IV-14

The upright E carries the micrometer eye-piece for measurement of width of bands. By turning the graduated screw-head M, the eye-piece with the vertical cross-wire, mounted on a movable frame, moves at right angles to the length of the bench. An auxiliary lens L of short focal length is mounted on the upright F.

**Adjustment of the optical bench.** Before taking readings the following adjustments must be made :

(1) **Making the slit and the cross-wire vertical.** For this purpose the eye-piece is focussed on the cross-wires. The upright with the auxiliary lens is placed between the eye-piece and the slit which is opened wide, so that, an illuminated plumb-line suspended on the side of the slit away from the eye-piece may be seen through the slit. By moving the auxiliary lens the image of plumb line is focussed on the cross-wire of the eye-piece and the cross-wire is rotated into the position where it is parallel to the plumb-line. One edge of the slit is then focussed on the cross-wire. The frame of the slit is then

rotated by the tangent screw till the edge of the slit is parallel to the vertical cross-wire. The slit, the auxiliary lens, and the eye-piece are then moved up or down till their centres are almost at the same height from the bed of the bench.

(2) **Adjustment of the biprism.** The upright carrying the auxiliary lens is then removed and the upright D carrying the biprism with its refracting edge nearly parallel to the slit is placed between the eye-piece and the slit. The frame carrying the biprism is moved up or down till the centre of the biprism is at the same height from the bed of the bench as the centre of the slit. The slit is now illuminated by white light and made narrow. The eye-piece is moved close to the biprism. If, on looking through the eyepiece, fringes are not observed, the upright carrying the biprism is moved perpendicularly to the length of the bench till the fringes appear in the field of view. The fringes thus observed would generally be indistinct. To make the fringes distinct, the biprism is rotated about a horizontal axis by the tangent screw provided till the fringes are very distinct. This will happen when the common base of the biprism is parallel to the length of the slit.

(3) **Making the apparatus co-axial.** Next look through the eye-piece and at the same time move it away from the biprism. If the fringe system appears to travel across the field of view, the plane passing through the slit and the common base of the biprism is not parallel to the length of the bench. To make this adjustment, illuminate the slit with white light and note the direction of movement of the fringes as the eye-piece is moved away from the bi-prism. Holding the eye-piece at a certain distance from the biprism, move the biprism at right angles to the length of the bench till the central white fringe coincides with the cross-wire. Next move the eye-piece close to the biprism and turn the micrometer head of the eye-piece till the central white fringe again coincides with the cross-wire. Again move the eye-piece away from the biprism and bring the white fringe into coincidence with the cross-wire by moving the biprism at right angles to the length of the bench. Then move the eye-piece again close to the biprism and move the cross-wire till it coincides with the white

fringe. *As a general rule, when the eye-piece is distant move the biprism transversely and when the eye-piece is close to the biprism move the eye-piece transversely to bring the white fringe on the cross-wire.*

The above procedure is repeated several times till there is no movement of the fringes relative to the cross-wire when the eye-piece is moved in the direction of the bench. The whole instrument would then be co-axial. The instrument is now ready for measurement.

**Determination of index error.** To determine the index error between the slit and the eye-piece, the eye-piece is placed between the biprism and the slit and the frame carrying the cross-wire of the eye-piece is removed. One end of a rod of known length (the index rod) is placed in contact with the centre of the slit and the eye-piece is slowly moved towards the rod, till the other end of the rod is sharply in focus. The readings of the index mark attached to the uprights carrying the slit and the eye-piece are then taken. The difference of the distance between these two readings and the length of the index rod gives the index error. Then put the frame carrying the cross-wires inside the eye-piece and adjust its distance from the eye-piece till it is sharply in focus.

**Determination of width of a fringe.** The slit is now illuminated with light whose wave-length is to be determined. The eye-piece is then set at a definite distance from the biprism, and its index reading is taken. The distance between every five fringes starting from one end of the fringe system is determined by turning the micrometer screw-head always in the same direction in order to avoid *back-lash error*. The measurement is repeated from the other end of the fringe system. Mean of several such readings gives the mean width of five fringes. This gives the value of $r$ (corresponding to $n=5$) in Eqn. (21·1). The difference of index readings of the slit and the eye piece corrected for the index error gives the value of $(b+c)$ in the same equation.

**Determination of distance (2d) between the virtual sources.** For this purpose the eye-piece is set at a distance greater than four times the focal length of the auxiliary lens which is now placed between the biprism and the eye-piece. By the displacement method (See Geometrical Optics P. 99) two images of the virtual sources are formed in the focal plane of the eye-piece for two positions of the auxiliary lens. For each position of the lens the distance between the two real images of the two virtual sources is measured by turning the micrometer screw-head. If $h_1$ and $h_2$ be these distances, then

$$2d = \sqrt{h_1 h_2} \text{ (See Geometrical Optics Eqn. 47·8)}$$

Since $2d$ is a very small quantity, it forms the largest source of error in the calculated result. Hence it must be measured as carefully as possible.

**22. Displacement of Fringes by a thin Plate** Let A and B (Fig. IV-15) be two coherent interfering sources of monochromatic light. Let a thin plate G of any transparent substance of thickness $t$ and of refractive index $n$ be placed in the path of the beam coming from A. The difference of air-paths from A and B to a point P on the screen is

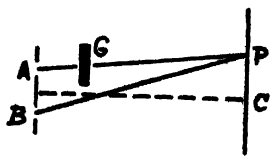



Fig. IV-15

$$\begin{aligned}\delta &= \text{BP} - [(\text{AP} - t) + nt] \\ &= \text{BP} - \text{AP} - (n-1)t\end{aligned}$$

or, $$\delta = \frac{2x_s d}{c} - (n-1)t \qquad .. \quad (22·1)$$

where $x_s$ is the distance of $s^{\text{th}}$ fringe at P by (Eqn. 17·2) from the centre of symmetry C, $2d$ is the distance between the sources A and B and $c$ is the distance of the screen CP from the line AB. If the fringe at P is bright, then

$$\frac{2x_s d}{c} - (n-1)t = s\lambda \qquad [s = 0, 1, 2, 3, 4 ..]$$

or, $$x_s = \left\{ s\lambda + (n-1)t \right\} . \frac{c}{2d} . \qquad .. \quad (22·2)$$

For the central band $s = 0$, so that

$$x_0 = \frac{(n-1)ct}{2d} \qquad .. \quad (22·3)$$

If $t$ were equal to zero, that is, if the plate were removed $x_0=0$ and the central fringe would coincide with the point C. Hence by interposition of the plate, the fringe system are displaced by a distance $(n-1)ct/2d$.

From observation of displacement of the central fringe the refractive index of material of the plate can be determined if the thickness $t$ of the plate be known and *conversely*. Fresnel and Arago used the method of displacement of the fringes for the determination of refractive index of gases. The method is very sensitive to changes of refractive index. Any change in refractive index due to variation of temperature of the order of $\frac{1}{10}$th of a degree centigrade can be easily detected by this method.

**Test of the theories of light.** From the displacement of fringes produced by the thin plate very conclusive proof can be obtained regarding the correctness of the corpuscular or the wave theory of light. If $v$ be the velocity of light in the medium of the plate and $v_0$ its velocity in the outer space, we have $n=v_0/v$ and

$$x_0=\frac{\left(\frac{v_0}{v}-1\right)ct}{2d}$$

If light moves faster inside the plate than outside, $v_0/v$ is less than 1 and, therefore $x_0$ is negative, that is, the central fringe moves below the point C. On the other hand, if $v$ is less than $v_0$, $v_0/v$ is greater than 1 and $x_0$ is positive, so that, the central fringe moves above the point C. Experiment shows that the central fringe always moves towards the side of the plate, so that, the velocity of light is smaller in the medium of the plate than in free space. Hence the experiment gives additional proof in support of the wave theory of light.

**23. Change of Phase on Reflection.** It is a well-known fact in acoustics that a wave of condensation moving in a rarer medium is reflected as a wave of condensation when it is incident on a denser medium. Similarly a wave of

rarefaction is reflected as such from the surface of a denser medium. But, if a wave of compression were initially moving in a denser medium, then, on encountering a rarer medium, it is reflected as a wave of rarefaction. In the former case, a change of phase angle equal to $\pi$ is introduced in the reflected wave, while in the latter case, the phase of the reflected wave is the same as that of the incident wave.

**Stoke's method of proof.** Light propagation being of the nature of wave motion, we expect a similar change of phase on reflection. An evidence of such a change we have already obtained in the case of formation of interference fringes with Lloyd's single mirror. Stoke's method of proof is based on the Schuster's principle which runs as follows:—

"*if at any time all velocities in a dynamical system are reversed and there is no dissipation of energy, then the whole previous motion is reversed. Any configuration of the system which existed at the time t before reversal took place, will, therefore, again exist at the time t after reversal.*"

Thus, let a portion of the ray AB incident at a point B (Fig. IV-16) on the surface of separation between two media with different optical densities be partly reflected in the direction BC and partly refracted in the direction BD. Let $a$ be the amplitude of the incident waves. Let $b$ and $c$ be the fractions of amplitude per unit amplitude of the incident light that are reflected and refracted respectively. Then the amplitude of light reflected in the direction BC is $ba$ and that refracted in the direction BD is $ca$.

By Schuster's principle, if the directions of the reflected and the refracted rays be reversed, we would get a ray in the direction BA of amplitude $a$ and nothing else. But the ray BC reversed would according to the laws of reflection and refraction give a reflected ray BA of amplitude $ab^2$ and a refracted ray in the direction BE of amplitude $abc$. Similarly, the ray BD reversed would give rise to a reflected ray in the direction BE of amplitude $ace$, the fraction $e$ being different from $b$, since, in this case, the direction of reflection is from the upper into

the lower medium. The ray BD reversed would also give a refracted ray in the direction BA of amplitude $acf$, $f$ being different from $c$ for the same reason. By Schuster's principle, the amplitude of the ray in the direction BE must be zero; so that

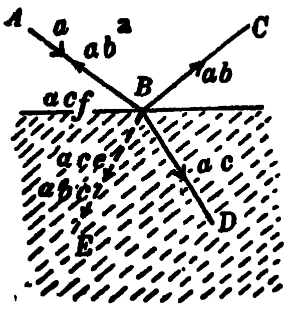



Fig. IV-16

$$ace+abc=0$$

or, $$b=-e.$$

By the same principle,

$$acf+ab^2=a$$

or, $$cf=1-b^2.$$

Since $b$ and $e$ are equal and of opposite signs, the two waves are in opposite phases and they destory one another by interference.

**24. Interference of rays reflected from a film :** Let AB and CD be the bounding surfaces of a film of refractive index $n$. A ray PQ striking the upper surface at Q at angle of incidence $i$ is partly reflected in the direction QT. The remaining portion is refracted into the film in the direction QR. At R a portion is refracted into air and the remaining portion is internally reflected in the direction RS. At S, a portion is refracted into air in the direction ST′ which is parallel to QT. The rays QT and ST′ are then combined together by a lens, so that, they may produce interference effects.

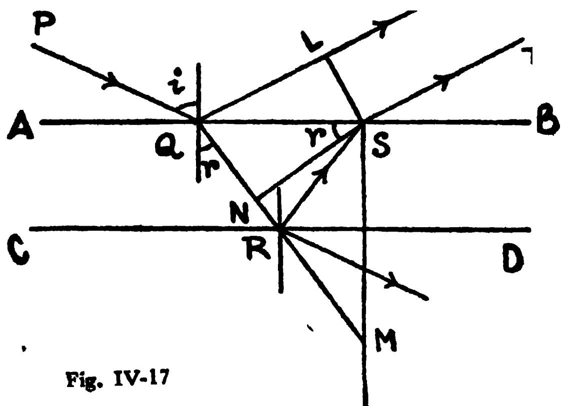



Fig. IV-17

**Calculation of the path difference :** Drop two perpendiculars SL and SN on QT and QR respectively. Produce QR

to M so that, QR=RM and join MS. Then MS is perpendicular to CD. The path difference between the two interfering rays QT nd ST′ is

$$\delta' = n(QR+RS)-QL = n(QN+NR+RS)-QL$$
$$= n(QN+NM)-QL$$

Now $QL = QS \sin i$; $nQN = nQS. \sin r = QS \sin i$

$\therefore \qquad QL = nQN$

Further $\qquad NM = MS. \cos r = 2e \cos r \qquad (24{\cdot}1)$

where $e$ is the thickness of the film. Hence the path difference is

$$\delta' = n.MN = 2ne \cos r \qquad \ldots$$

Since the two reflections at Q and R occur under opposite conditions, a further difference of path $\lambda/2$ is introduced between the two interfering rays. Thus, the actual path difference is

$$\delta' = 2ne \cos r - \lambda/2 \qquad \ldots \quad (24.2)$$

**Colour of thin films.** In the case of a film bounded by parallel faces, if the thickness $e$ of the film is very small, then the phase angle

$$\delta = \frac{2\pi}{\lambda}.\ 2ne \cos r - \lambda/2$$

is not appreciably altered by the possible variations in the values of $r$ (or $i$). Hence the intensity is not affected by variation of angle of incidence, so that, the film appears to be of uniform intensity throughout. If the film is viewed with white light, some of the colours would be destroyed by interference and the film would appear to be of uniform colour throughout. If the film is thick, fringes would appear, since, in this case, $e \cos r$ would vary periodically.

**25. Newton's Rings.** As mentioned in the Introductory Chapter, Newton obtained a number of circular fringes round the point of contact of a long-focus convex lens with a plane sheet of glass. To produce Newton's rings, the experimental arrangement shown in Fig. IV-18 may be conveniently used.

**The apparatus.** Light from a sodium flame is rendered parallel by a converging lens L. It then falls on a glass plate G. inclined at angle of 45° to the incident rays. A portion of the

light is reflected vertically downwards into the air-film between the lens and the plate glass. Some of these rays are then reflected upwards from the upper and lower surfaces of the air film. They then suffer deviation of 90° after reflection at a total reflection prism P. These rays are then received by the reading telescope T.

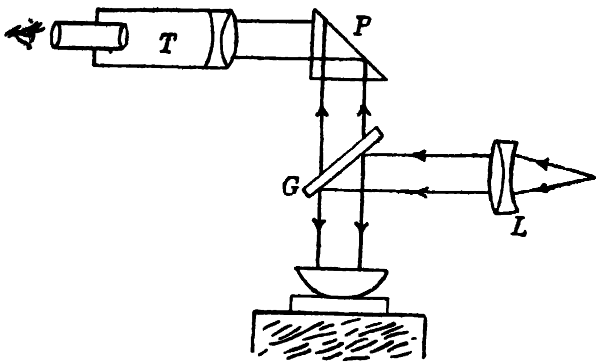



Fig. IV-18

**Calculation of path-difference.** In Fig. IV-19, let POQ be the section of the air-film enclosed between the surface OP of a plate glass and a lens of which OQM is the circle of curvature. A vertical section of the air-film by the plane of the paper is wedge-shaped. Since the lens is of very large radius of curvature the path-difference between the externally and internally reflected rays from the film at Q is by Eqn. (24·1)

$$\delta' = 2ne \cos r \qquad .. \quad (25{\cdot}1)$$

where $n$ is the refractive index of the film and $e(=PQ)$ is the thickness of the film at the point Q (Fig. IV-19). If $\rho$=the distance OP (O being the point of contact) then, by geometry

$$\rho^2 = 2Re,$$

$R$ being the radius of curvature of the lens. Hence, substituting for $e$ in Eqn. (25·1), we get

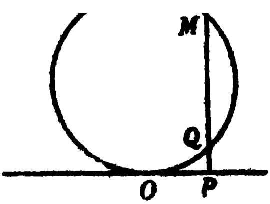



Fig. IV-19

$$\delta' = \frac{n\rho^2}{R} \cos r. \qquad ... \quad (25{\cdot}2)$$

If the figure IV-9 be rotated about the diameter of the circle passing through the point O then the point P would describe a circle of radius P round the point O. At every point on this circle the thickness $e$ of the film would be the same. Hence the path-difference between the two interfering rays would be the same on the circle. So the fringes are concentric circle round the point of contact O. They would be alternately bright and dark. These circles are called Newton's rings. Substituting the value of $\delta'$ in Eqn. (24·2) and the radius $\rho_s$ of the $s$th bright ring

$$\rho_s^2{}_{(bright)}=\frac{(2s-1)\lambda R}{2n\cos r} \quad \dots \quad (25\cdot3)$$

similarly $$\rho_s^2{}_{(dark)}=\frac{2s\lambda R}{2n\cos r} \quad \dots \quad (25\cdot4)$$

**Characteristics of Newton's fringes.** To observe Newton's rings a wide source of light must be used. The central fringe corresponds to the value of $s=0$. In the case of a convex lens and a plate placed in contact, $e=0$ at the point of contact. Since $\rho^2=2Re$, this makes $\rho=0$, so that the central ring is dark. The path-difference being zero at the point of contact, the neutralisation is effected by the path-difference $\lambda/2$ introduced by reflection under opposite conditions. This proves the correctness of the results of Art 23.

The system of fringes lie within the film and, to observe the fringes, the eye or the telescope must be focussed on the film.

The radius of the successive rings are proportional to the square root of natural numbers, those of dark rings being proportional to the square root of the even numbers and those of the bright rings to square root of the odd numbers.

By pressing the lens on the plane surfaee, the radius of curvature of the lens at the point of contact is increased and the rings move away from the centre. The rings would contract or move towards the centre, if the refractive index of the film is increased. Thus by putting a transparent oil in between the lens and the plate, the rings are found to move towards the centre.

**Width of a fringe.** It follows from Eqn. (25 4), that the radius of the $s$th dark ring is

$$\rho_s^2 = \frac{2sR\lambda}{2n \cos r}$$

and that of the $(s+1)$th dark ring is

$$\rho^2_{s+1} = \frac{2(s+1)R\lambda}{2n \cos r}$$

Therefore, $$\rho^2_{s+1} - \rho_s^2 = \frac{R\lambda}{n \cos r}.$$

But $\rho^2_{s+1} - \rho_s^2 = (\rho_{s+1} - \rho_s)(\rho_{s+1} + \rho_s) = (\rho_{s+1} + \rho_s).2\rho_s$ to a high order of approximation.

Hence $$\rho_{s+1} - \rho_s = \frac{R\lambda}{n \cos r.\, 2\rho_s}$$

Hence the fringe becomes narrower as the diameter of the fringe increases.

Since the radius of a fringe varies as the square root of the wave-length, the radius of the red fringe is greater than that of the violet. If the fringes are formed with white light the edge of a ring nearer the point of contact would be violet and the outer edge would be red. Due to overlapping of colours only a small number of fringes would be formed and the system would gradually merge into general illumination.

**Rings by transmitted light.** If the light incident on the film is white the two systems of fringes formed by transmitted and reflected light are complementary to one another. The bright fringes on a dark background observed by reflected light appear as dark fringes on a lighted background observed by transmitted light.

The complementary nature of the transmission and reflection rings was established by Arago in the following way. He placed the plate and the lens in vertical position over a uniformly illuminated sheet of white paper as shown in Fig. IV-20. An eye placed in the position shown in the figure would receive both

reflection and the transmission rings simultaneously. These rings being of complementary character, they would combine to produce uniform illumination.

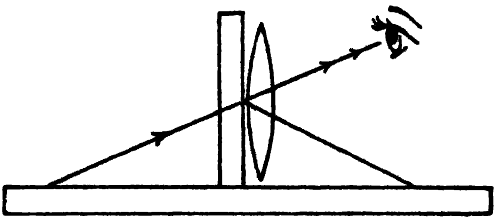


Fig. IV-20

**Test of theories of light.** It is found that the radius of a particular ring formed from a film of water enclosed between the lens and the plate is smaller than that formed from a film of air enclosed. Hence, it follows from Eqn. (25·2) that $n$ for water must be greater than that for air. This is in accord with the results of observation.

**Determination of wave-length of light.** For measurement of wave-length of light with the help of Newton's rings an air film between a convex and a plane surface is generally used. If the incidence on the film is normal as in Fig.IV-18 and if observation is made in the region close to the point of contact, then $i=0$, and $r=0$, so that, putting $n=1$ for air, we have for the radius of the $s$th dark ring

$$\rho_s^2 = sR\lambda.$$

From this equation, knowing $\rho_s$, $s$, and $R$, $\lambda$ can be calculated. Though apparently simple, this method does not take into account certain possible sources of error. These are the following :

(1) Due to pressure exerted by the lens on the plate, the curvature at the point of contact may be altered.

(2) It is uncertain whether the lens and the plate are actually in contact. Unless the surfaces near the point of contact are perfectly clean, they remain separated by a distance, however small. Due to these causes there always remains some uncertainty

regarding the order $s$ of the ring being measured. To get rid of this difficulty the diameter of the $(s+n)$ ring is also measured. Supposing that the lens and the plate are separated by a small distance $d$, we have by putting $n=1$, and $r=0$ in Eqn. (25.5)

$$\rho_s^2 = sR\lambda - 2d.R.$$

so that $$\rho^2_{s+n} = (s+n)R\lambda - 2dR..$$

Hence $$\rho^2_{s+n} - \rho_s^2 = nR\lambda. \qquad (25\cdot11)$$

from which $\lambda$ can be calculated.

**Measurement of small curvature.** The measurement of $\lambda$ by the above method involves the measurement of $R$ which is generally large. This measurement of small curvature can be effected with the help of Abbe's spherometer. A sketch of the apparatus is shown in Fig. IV-21. It consists of a ring $R$ carefully ground to the shape shown in the inset. The surface whose curvature is to be measured is placed on the ring. If the surface is concave it would rest on the outer rim and if it is convex, it would rest on the inner rim. To measure the radius of curvature, a plane glass plate is first placed on the ring and a plunger L concentric with the ring and carrying a scale is slowly raised by a counter-weight (C.W) till it just touches the plane surface. The position of contact between the plunger and the surface

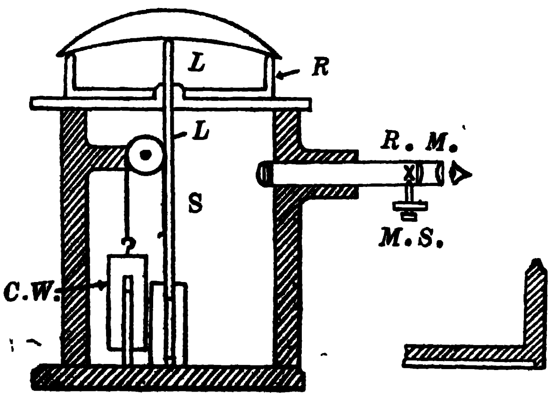



Fig. IV-21

of the plate can be accurately determined from the appearance of interference fringes round the point of contact. The reading of the scale on the plunger is then taken by means of a

reading microscope. The plane sheet is then replaced by the curved surface and the same process is repeated. If $h$ is the difference between the two readings and $r$ is the radius of the ring, then the radius of curvature $R$ is calculated from the relation

$$R=\frac{r^2}{6h}+\frac{h}{2}.$$

**26. Uses of Interference Fringes.** Production of Newton's rings provides very sensitive and accurate method of optical testing in certain cases which we now proceed to describe.

**Testing glass plates for flatness and plane parallelism.** The apparatus used for this purpose is represented diagrammatically in Fig. IV-22. The plate to be tested is placed on a base and test glass which is truly plane parallel is placed on the surface to be tested. Light from a mercury arc lamp S, filtered through green glass plate D is incident on a plate glass M at an angle of 45° and then falls normally on the test plate. The normally reflected rays from the surface of contact are received by an eye placed vertically above the test plate. If the surface to be tested coincides at all points with the surface of the test plate, then the field would be completely dark. On account of the risk of scratching the surfaces in contact, this procedure is seldom used. Instead, the two surfaces are placed in contact over a line thus enclosing a wedge-shaped film between them. If the surfaces are plane, this will give rise to a system of alternately bright and dark straight fringes parallel to the line of contact (Fig. IV-23 $a$).

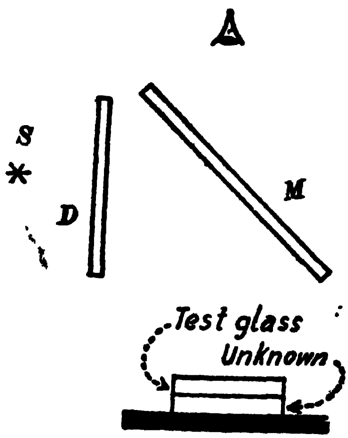


Fig. IV-22

If the surface to be tested is convex, perfectly concentric Newton's rings would appear round every point of contact as the

convex surface is rocked about on the surface of the test plate (Fig. IV-23 b). A concave surface would also give similar result (Fig. IV-23 c) except for the obvious differences arising from the edges being in contact instead of the centre as in the case of a convex surface. If the surface to be tested is irregular, distorted Newton's rings would appear around every point of nearest approach as shown in Fig. IV-23 *d*.

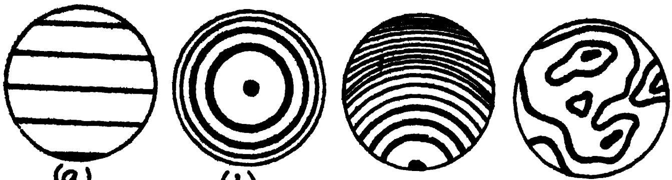



Fig. IV-23

**Comparison of thickness.** In the construction of standardised machine parts accurate measurement of dimensions of the parts are often needed. These measurements can be effected with the help of interference fringes. Fig. IV-27, illustrates the method of measurement of length of a block with the help of one standard gauge and a test-plate. The block and the gauge are placed together on a base plate in closest contact and the test plate is placed upon them as shown in Fig. IV-27. If the two lengths are not exactly equal, a wedge of air-film would be enclosed under the test plate. By counting the number of interference fringes formed, the difference in length can be determined. For, let $e$ be the difference in length, then the order of the fringe at the base of the wedge is obtained from the relation

$$2ne \cos r = s\lambda.$$

By putting $n=1$, and $\cos r=1$, (since the incidence on the film is normal) this gives

$$e = \frac{s\lambda}{2}. \qquad \cdots \quad (26\cdot1)$$

Hence, by counting the number of fringes (leaving the one at the apex), the difference in thickness can be expressed in terms of the number of half wave-lengths.

The method of measurement of diameter of a cylindrical body by the above method is illustrated in Fig. IV-28.

Measurement of thermal expansion, magnetostriction, coefficients of elasticity such as Young's modulus, Poisson's ratio, where small changes of length are involved can be very accu-

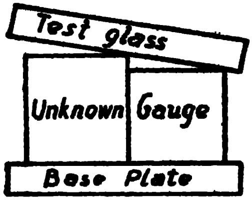



Fig. IV-27

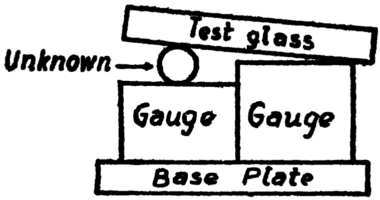



Fig. IV-28

rately measured by observation of interference fringes between a plane and a sphere. For this purpose the plane is attached to the expanding edge of the body under test and the sphere to the end of the micrometer screw. As the separation between the plane and the sphere alters, the fringes close in at the centre, bright rings and dark rings interchanging their positions for every change in separation by a wave-length.

In certain types of three legged spherometers used for accurate measurement, a glass ball is attached to the end of the screw to determine the point of contact of the screw with the base plate or the curved surface under test. When the two surfaces touch one another, Newton's rings appear around the point of contact.

**Measurement of small angles.** Small angles can also be measured by interference methods. Using Eqn. (26.1)

$$e=\frac{s\lambda}{2},$$

if $x$ be the distance between the apex and the base of the air-wedge, as in Fig. IV-27, the angle $a$ of the wedge is

$$a=\frac{e}{x}=\frac{s.\lambda}{x\,2}.$$

In other words, the angle of the wedge is equal to the number of bands per unit length into half the wave-length of the light used.

**Structure of the sodium line.** In 1862 Fizeau observed by separating the distance between the lens and the plate that at the 500th order, Newton's rings with sodium light became very indistinct and they again became very distinct at the 1000th order. This observation was the first indication of the fact that the spectral lines are usually of a multiplet structure, that is, they consist of two or more homogeneous lights of slightly different wave-lengths. This observation leads to a simple method of determining the difference of the two wave-lengths in the beam.

Since the width of a dark band is equal to that of the next bright band, the fringes become indistinct when the maximum of a wave-length $\lambda_1$ of $s$th order falls on the minimum of wave-length $\lambda_2$ of $(s-1)$th order. Hence, if $\rho_s$ is the diameter of the $s$th bright ring of wave-length $\lambda_1$, then, for disappearance of fringes $\rho_s$ must also be the diameter of the $(s-1)$th dark ring of wave-length $\lambda_2$. Hence from Eqns. (34.3) and (34.4) we obtain,

$$\rho_s^2=\frac{(2s-1)\lambda_1 R}{2\mu\cos r}-2dR=\frac{(2s-1)\lambda_2 R}{2\mu\cos r}-2dR,$$

or, $$(2s-1)\lambda_1=(2s-1)\lambda_2$$

or, $$\frac{\lambda_2-\lambda_1}{\lambda_2}=\frac{1}{2s-1}$$

or, $$\lambda_2-\lambda_1=\frac{\lambda_2}{2s-1}$$

Taking $\lambda_2=5890\times10^{-8}$ cm., $\lambda_2-\lambda_1=\dfrac{5890\times10^{-8}}{2\times500-1}=5{\cdot}98\times10^{-8}$

Hence, the yellow sodium line consists of a doublet differing in wave-lengths by about $6\,\mathring{A}$ ($1\,\mathring{A}=10^{-8}$ cm.)

**27. Stationary Light Waves.** It has been proved in Chapter II that two trains of similar waves moving simultaneously in opposite directions over the same part of an elastic medium combine together to produce a train of stationary waves. In this train of waves a number of points separated by distances equal to half the wave-

length of the component waves and lying in the direction of propagation are permanently at rest. These stationary points are called nodes. From one node to the next the amplitude of vibration gradually increases, reaching maximum value at points midway between them and then falls to zero value again at the next node. These points of maximum amplitude of vibration are called antinodes.

From the view-point of acoustics nodes are points of silence and antinodes are points of maximum loudness of sound. Since light is wave-motion in ether, they would also form stationary waves, the nodes corresponding to points of darkness and antinodes to points of maximum illumination.

In the year 1890 Wiener demonstrated the existence of stationary light waves by reflection from a mirror. To record the statinoary waves, a thin film of photographic emulsion was spread on a mirror surface, the exposed film surface being inclined at a small angle to the surface of the mirror. He then allowed a beam of monochromatic light to be incident normally on the mirror through the emulsion. Since the reflected waves were similar to the incident waves (except for a slight loss of

Fig. IV-29

amplitude by reflection), stationary waves were formed within the emulsion by superposition of the incident and reflected waves. Over the nodal planes there was no photographic action and over the antinodal planes, the action was very intense. Hence on developing the film after exposure, there appeared in the film a number of parallel layers which were alternately slightly dark and transparent, the dark layers corresponding to antinodal planes and the transparent layers to nodal planes.

A micro-photograph of a section of the film is reproduced in Fig. 42.

**Lippmann's colour photography.** To understand the principle of Lippmann's colour photography, let us suppose that the wave-length of monochromatic light falling on the film was $\lambda$. If the same light is incident on the developed plate, the path-difference of light reflected from two consecutive layers would be equal to a complete wave-length, so that light reflected from these layers would agree in phase and thus reflected beam would be bright. In other words, the film after developing would totally reflect the light to which it was exposed originally. Any other wave-length incident on the film would pass through the film and would be absorbed completely if it is backed by a black varnish. If the film were exposed to a composite beam of light, nodal and antinodal planes corresponding to every wave-length in the composite beam would be produced within the emulsion (Fig. VI-30). If such a film is exposed to white light, it would selectively reflect those colours to which it was originally exposed and absorb the rest, so that, the image would be seen in its original colours.

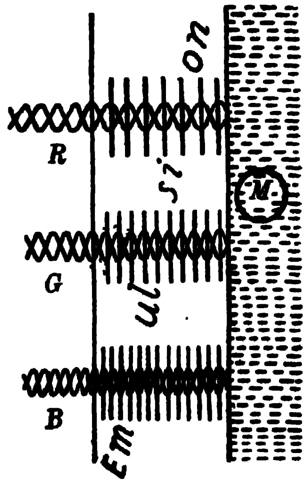


Fig. IV-30

The plates required for colour photography are prepared by a special process, since the grains of ordinary photographic plate are too coarse to be of any use. The plate is placed in the camera, the glass side facing the lens and the film side backed by a layer of mercury to reflect the light into the emulsion. The process of taking photograph by this method is very complicated and hence it has little commercial value. Lippmann, however, took a number of photographs by this method and was able to reproduce prismatic spectra in their natural colours.

# CHAPTER V

## DIFFRACTION OF LIGHT

**Interference and diffraction.** In the chapter on interference we have discussed phenomena exhibited by superposition of rays coming originally from the same point-source. According to Huygen's principle each point on a wave front is a source of secondary wavelets. A wave-front is the locus of all points which are in the same phase of vibration at an instant. Hence the secondary sources lying on a wave-front are coherent sources, that is, the rays coming from such secondary sources can interfere with each other. The interference of secondary wavelets derived from a particular wave-front is responsible for another class of phenomena which we call diffraction.

**28. Modified Huygens' Principle. Huygens' assumptions.** Following Huygens (See Introductory Chapter, Art. 2) the effect of secondary wavelets derived from a particular wave-front was supposed to be confined within the envelope of the secondary waves, that is, to the tangent plane touching the secondary wavelets. This assumption was made in order to explain the so-called rectilinear propagation of light. This assumption gave rise to the following difficulties :

(1) It ignored the diffraction or bending of light round the edges of an obstacle which is actually observed.

(2) If each point of a wave-front is the centre of another wave, the latter would necessarily radiate light energy in all possible directions ; in other words, even in a homogeneous medium, there would be simultaneous propagation of waves from each point of the medium traversed by light in the backward as well as in the forward direction. This would mean that at every point in a homogeneous medium there would be reflection and refraction of light, which is contrary to experience.

(3) Huygens considered light waves to be longitudinal. Hence he could not offer any explanation of the phenomenon of polarisation of lightt.

**Fresnel's modification.** Fresnel assumed that the light vibrations were transverse, so that, the phenomenon of polarisation was a necessary consequence.

Instead of assuming arbitrarily that the effect of secondary wavelets was confined to the envelope only, he took a more rational view of the subject. He used the principle of superposition to obtain the resultant effect of the secondary waves at a particular point of the medium traversed by the waves. To obtain the resultant displacement by the principle of superposition, it is necessary to pay due attention to the following points :

(1) **The phase of vibration** of each wavelet reaching the point under consideration at an instant. This, of course, depends on the differences of paths connecting the point and the secondary sources.

(2) **Obliquity of the rays.** This is measured by the angle which the ray makes with the normal to the wave-front at the centre of the secondary wave. In considering the question of obliquity, Fresnel assumed the result of Stokes' calculation who proved mathematically that the intensity in a direction $\theta$ with the normal varies as $(1+\cos\theta)$. Hence, the intensity is zero, in the direction $\theta=\pi$, that is, in a direction opposite to the direction of propagation. This partly meets the objection (2) stated above.

(3) **The amplitude.** As the secondary spherical wave expands, the energy is distributed over larger and larger areas. Since the intensity of light falls off inversely as the square of the distance and since the intensity is proportional to the square of amplitue the amplitude of vibration would fall off inversely as distance.

**29. Calculation of Intensity.** We shall now illustrate Fresnel's ideas by calculating the intensity at a point due to wavelets originating from a plane wave-front. Let ABCD be a plane wave-front moving in the direction shown by the arrowheads (Fig. V-I).

To determine the resultant displacement at the point P due to secondary disturbances sent out by every point on the wave-front

ABCD, we drop a perpendicular from P on the wave-front. The foot O of the perpendicular is called the *pole* with respect to P. Instead of summing up the displacement from every point of the wave-front taken at random, we divide the wave-front into small areas or zones in the following way for convenience of calculation :

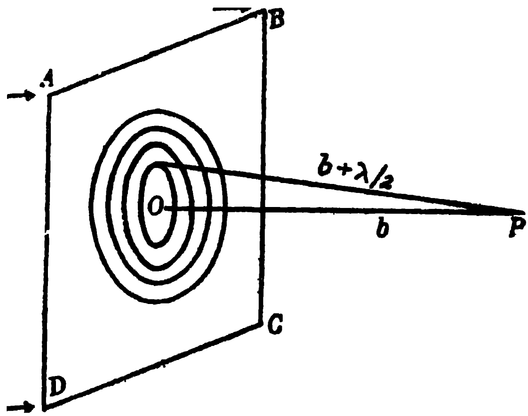



Fig. V-1

**Zones or half-period elements.** Let the distance OP (Fig. V-1) be equal to $b$. Let $\lambda$ be the wave-length of the light wave. With P as centre and radii equal to $(b + \lambda/2)$, $(b+2\lambda/2)$, $(b+3\lambda/2)$, etc. we describe concentric spheres. These spheres would intersect the plane wave-front in concentric circles with O as common centre. Since the radii of the inner and outer circles bounding a particular zone differ by $\lambda/2$ (or by half period), these zones or areas bounded by two successive concentric circles are also called *half-period elements.* Since wave-length of light is very small, the area of zones is also very small. Hence obliquity may be supposed to remain constant from the inner to the outer edge of one zone. For the same reason the change of amplitude due to change of distance may also neglected for different points of the same zone.

**Area of a zone.** Let the radius of the outer circle bounding the $n^{th}$ zone $(X_{n-1}X_n)$ be $R_n$ (Fig. V-2). Then the area of the

$n^{th}$ zone is $2\pi R_n.(X_{n-1}X_n)$. From the point $X_{n-1}$ drop a perpendicular on $PX_n$ cutting it at N.

From the similar triangles $(X_nX_{n-1}N)$ and $OPX_n$,

$$\frac{X_nX_{n-1}}{X_nN}=\frac{PX_n}{OX_n}$$

or putting $PX_n = r_n$

$$\frac{X_nX_{n-1}}{\lambda/2}=\frac{r_n}{R_n}$$

Hence the area of the $n^{th}$ zone is

$$A_n=2\pi R_n.\frac{r_n}{R_n}.\frac{\lambda}{2}=\pi r_n\lambda$$

Thus the area of a zone increases as we move from the pole outwards.

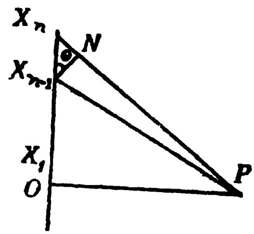


Fig. V-2

To form an idea of the size of a zone, let us calculate the radius of the first zone for $b=20$ cm. Putting $r_1=b=20$ cms. and $\lambda=5\times10^{-5}$ cm.

$$A_1=\pi r_1\lambda=\pi\times20\times5\times10^{-5}.$$

But $\quad A_1=\pi R_1^2$

where $R_1$ is the radius of the first zone. Hence

$$\pi R_1^2=\pi\times20\times5\times10^{-5}$$

or $\quad R_1^2=20\times5\times10^{-5}$

or $\quad R_1=3{\cdot}16\times10^{-2}\text{cm.}={\cdot}316 \text{ mm.}$

**Calculation of intensity.** Since the secondary sources are distributed uniformly over the wave-front, the displacement at P due to disturbances coming from a particular zone would be proportional to the area of the zone. Hence the effect on the amplitude due to *area* of the $n^{th}$ zone is

$$s_n \propto \pi r_n\lambda.$$

Considering the question of distance, we have seen that the amplitude varies inversely as the distance $r_n$. Hence the combined effect on the amplitude due to *area and distance* is

$$s_n \propto \pi r_n\lambda.\frac{1}{r_n}$$

or $\quad s_n \propto \pi\lambda.$

It is, therefore, the same for all zones, since it is independent of $n$.

But the *obliquity* of a ray increases from the pole outwards. Hence due to this cause, the value of $s_n$ would steadily diminish from the pole outwards.

There is yet the question of *phase* to be considered. Since from one zone to the next, the path-difference changes by $\lambda/2$ the phase would change by $\pi$ from one zone to the next. This means that the amplitude of disturbance coming from successive zones would be alternately positive and negative.

Hence considering the effects of *area, distance, obliquity and phase*, the resultant amplitude at P (Fig. V-1) may be represented by the series

$$S = s_1 - s_2 + s_3 - s_4 + s_5 + \ldots\ldots \quad \ldots \qquad (29\cdot1)$$

where $s_1$, $s_2$, $s_3$, $s_4$ etc, are amplitudes due to the first, second, third, fourth etc. zones, $s_1$, $s_2$, $s_3$, $s_4$ etc. being in the diminishing order of magnitude.

**Summation of the series.** (*a*) **Algebraic method.** We can write the above series (29·1) of $n$ terms for resultant amplitude in the following form :

$$S = \frac{s_1}{2} + \left(\frac{s_1}{2} - s_2 + \frac{s_3}{2}\right) + \left(\frac{s_3}{2} - s_4 + \frac{s_5}{2}\right) + \ldots$$

$$+ \left(\frac{s_{n-2}}{2} - s_{n-1} + \frac{s_n}{2}\right) + \frac{s_n}{2}$$

The term within bracket such as $\left(\frac{s_{n-2}}{2} - s_{n-1} + \frac{s_n}{2}\right)$ is really the difference of arithmetic mean of the $(n-2)^{th}$ and $n^{th}$ term minus the $(n-1)^{th}$ term. Since each term of the series is less than the preceeding and greater than the succeeding and since they differ from each other by very small amount, the terms within bracket may with fair degree of accuracy be taken to be zero, so that

$$S = \frac{s_1}{2} + \frac{s_n}{2}$$

If $n$ is very large, $s_n$ is very small, and

$$S = \frac{s_1}{2}$$

Hence the total amplitude due to all zones is only half that due to the first zone.

(*b*) **Graphical method.** The same result can be obtained graphically as follows : Draw a line $OX_n$ (Fig. v-3) and divide $OX_n$ into $n$ equal segments at the points $X_1$, $X_2$, $X_3$, $X_n$ and from each of the division points erect ordinates of length $s_1$, $s_2$, $s_3$ etc. Writing the series in the form

$$S=(s_1-s_2)+(s_3-s_4)+(s_5-s_6)\ldots$$

we find that the successive terms inside bracket are represented in the figure by lengths $B_1b_1$, $B_2b_2$, $B_3b_3$, and so on. Hence

$$S=B_1\,b_1+B_2\,b_2+B_3\,b_3\ldots$$

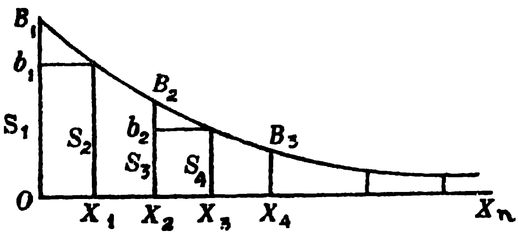



Fig. V-3

This sum (since the alternate differences are missing in the figure) is evidently equal to half of $OB_1$, that is, equal to $\frac{1}{2}s$.

(*c*) **Geometrical method.** If we divide any zone into a very large number of smaller zones, then the amplitude from the elementary zones would diminish very slowly from the inner to

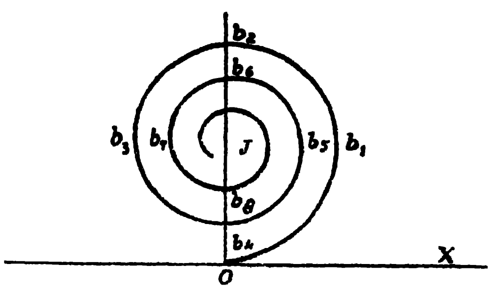



Fig. V-4

the outer edge of a zone. At the same time the phase will increase gradually by $\pi$ from the inner to the outer edge. If we

add the amplitudes from the elementary zones of the first half-period element by the geometric method the result will be represented by the curve $Ob_1b_2$ in Fig. V-4. The tangent to the curve at $b_2$ is obviously parallel to OX, since the phase at $b_2$ is $\pi$. Similarly the amplitude curve for the second zone would be represented by $b_2b_3b_4$, the tangent to the curve at $b_4$ being again parallel to OX. For the same reason, the amplitude curves for the third, fourth, etc. zones would be represented by $b_4b_5b_6$, $b_6b_7b_8$ etc. The curve for the disturbance from the zones would thus be represented by a flat sprial which gradually winds up towards the centre J. Since the resultant amplitude for any number of zons is obtaine by joining the end-points of the vibration curve, the resultant amplitude is evidently equal to OJ, which is only half that of $Ob_2$, the amplitude due to the first zone.

**Effect of aperture.** Suppose the incident wave-front ABCD in Fig. V-1 falls upon an opaque diaphragm with an adjustable aperture at the centre. If the diameter of the aperture is equal to that of the first zone, then the whole of the vibration curve is Fig. V-4 except the portion $Ob_1b_2$ is stopped out. Hence the amplitude at P in Fig V-1 is equal to $Ob_2$ which is double that of OJ. Hence the intensity at P would be four times the intensity if all the zones were exposed. If the diameter of the aperture is equal to that of the first two zones, the exposed vibration curve would be represented by $Ob_1b_2b_3b_4$ and the resultant amplitude would be represented by $Ob_4$ which is very small. Hence the intensity at P due to first two exposed zones would be indefinitely small. Similarly, if the first three zones were exposed, the amplitude at P would be represented by $Ob_6$ which is larger than OJ. Hence as the diameter of the aperture is gradually increased, the intensity at P passes alternately through maximum and minimum values.

*The intensity at P is maximum or minimum according as the number of exposed zones is odd or even.*

**Effects of screens.** Instead of an aperture, let us now place an opaque obstacle of adjustable diameter at the pole. If the diameter of the screen is equal to that of the first zone, then the

portion $Ob_1b_2$ only of the vibration curve (Fig. V-4) is stopped out. The resultant amplitude due to the remaining exposed zones is evidently, equal to $b_2J$ which is nearly half of $b_2b_4$ that is, half the first exposed zone. If the first two zones are cut off the resultant amplitude would be $b_8J$, that is, nearly half that due to third zone. Since the amplitudes of vibration due to successive zones diminish gradually, the intensity at P would gradually fall off as the diameter of the screen is increased. So there would always be some amount of light at the centre of the geometrical shadow. This is shown in Fig. 5 Plate VI which gives the photograph of shadow of a small opaque disc placed in the path of light. The photograph shows a bright spot at the centre of the shadow.

**Fresnel's theory and rectilinear propagation of light.** According to Fresnel-Huygen theory, the centre of the geometrical shadow due to an opaque circular disc is not dark particulary when the size of the obstacle is small. Arago and Fresnel actually observed a bright spot at the centre of the shadow. (Fig. 5 Plate VI). This bending of light rays round the edges of an obstacle so as to meet at the centre of its geometrical shadow is generally regarded as violation of one of the fundamental postulates of geometrical optics—that light travels in straight lines in homogeneous medium. In other words, Gemetrical Optics regards rectilinear propagation of light as fundamental and diffraction as only deviation. The truth is that diffraction is fundamental property of light and rectilinear propagation is only a special case which occurs when the size of the wave-front is unlimited or the wave length is large.

**30. Theory of Zone Plates** Since the alternate terms of the series (Eqn 29·1)

$$S = s_1 - s_2 + s_3 - s_4 + s_5 - s_6 + \dots$$

are of opposite signs, if the second, fourth, sixth etc. terms are removed then the resultant intensity would consist of terms with positive signs only, that is,

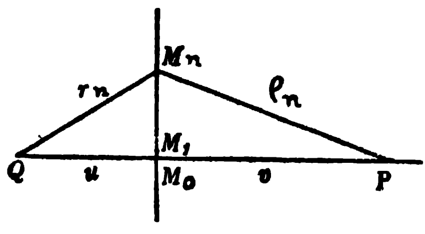
$$S = s_1 + s_3 + s_5 + s_7 + \dots$$

Accordingly, the resultant intensity at P (Fig. V-1) would be very large compared to the intensity when all the terms are present. This is the underlying principle of construction of the zone plate.

To construct a zone plate, circles are drawn on paper round a common centre with radii proportional to square root of natural numbers (see next paragraph). In this way the plane of the paper is divided into a number of zones. The alternate zones are blackened and a reduced photograph of the plate is taken. If the photographic plate thus prepared is placed normally in the path of a parallel beam of light, the rays transmitted through the plate would be brought to a sharp focus on the axis of the plate. A zone plate thus behaves like a converging lens and it thus gives strong support to Huygen-Fresnel theory of diffraction.

**Radius of the $n^{th}$ zone.** To calculate the radii of the successive zones, let us place a luminous point Q (Fig. V-5) on the axis of the zone plate $M_0M_n$, and let the distance of Q from the plate be $QM_0=u$. Let the distance of P from the plate be $PM_0=v$, where P is the image of Q. Also let the first, sceond,

Fig. V-5




third etc. zonal circles cut the plane of the zone plate at the points $M_1$, $M_2$, $M_3$ etc. respectively such that

$$\left.\begin{aligned}
QM_1+M_1P&=QM_0+M_0P+\frac{\lambda}{2}=u+v+\frac{\lambda}{2}\\
QM_2+M_2P&=QM_0+M_0P+\frac{2\lambda}{2}=u+v+\frac{2\lambda}{2}\\
QM_3+M_3P&=QM_0+M_0P+\frac{3\lambda}{2}=u+v+\frac{3\lambda}{2}\\
QM_n+M_nP&=QM_0+M_0P+\frac{n\lambda}{2}=u+v+\frac{n\lambda}{2}
\end{aligned}\right\} \quad \cdots (30{\cdot}1)$$

Let the radius of the $n^{th}$ zonal circle be $M_0M_n = R_n$. Let $QM_n = r_n$ and $PM_n = \rho_n$. Then

$$R_n^2 = r_n^2 - u^2 = (r_n + u)(r_n - u) = 2u(r_n - u) \text{ (approx.)}$$

or
$$r_n = \frac{R_n^2}{2u} + u.$$

Similarly,
$$\rho_n = \frac{R_n^2}{2v} + v$$

$$r_n + \rho_n = \frac{R_n^2}{2}\left(\frac{1}{u} + \frac{1}{v}\right) + \left(u + v\right) \quad \dots \quad (30.2)$$

But from Eqnl (30·1)

$$r_n + \rho_n = \left(u + v\right) + n\frac{\lambda}{2}. \quad \dots \quad (30·3)$$

Hence from Eqns. (30·2) and (30·3)

$$\frac{R_n^2}{2}\left(\frac{1}{u} + \frac{1}{v}\right) = \frac{n\lambda}{2} \quad \dots \quad (30·4)$$

or
$$R_n^2 = \left(\frac{uv}{u+v}\right)n\lambda. \ [n = 1, 2, 3, 4,] \quad \dots \quad (30·5)$$

Thus, the radius of the *nth* zone is proportional to $\sqrt{n}$ for given values of $u$ and $v$.

**The area of nth zone.** The area of the *nth* zone is

$$A_n = \pi(R_n^2 - R_{n-1}^2)$$
$$= \pi\frac{uv}{u+v}\left[n\lambda - (n-1)\lambda\right]$$

or
$$A_n = \pi\frac{uv}{u+v}.\lambda.$$

Hence all the zones are of equal area.

**Focal length of the zone-plate.** As already mentioned, if the alternate zones are blackened, there would be strong concentration of the rays at the point P. To find the focal length of the equivalent converging lens, we have from Eqn. 30·4

$$\frac{1}{v} + \frac{1}{u} = \frac{n\lambda}{R_n^2}. \quad \dots \quad (30·6)$$

Comparing with the expression for focal length of a converging lens, the focal length is given by

$$\frac{1}{f_n} = \frac{n\lambda}{R_n^2} \quad \dots \quad (30·7)$$

There is, however, an important point of difference between a zone plate and a converging lens. In the case of a converging lens, there is one point-image for one point-object. In the case of a zone plate, there is a large number of point-images of gradually diminishing intensity lying between the pole and the brightest image given by Eqn. (30·7) These arise for those values of $v$ for which each of the zones contains an odd number of new zones. Positions of the corresponding subsidiary foci are given by

$$f_1=\frac{R_n^2}{n\lambda}, f_2=\frac{R_n^2}{3n\lambda}, f_3=\frac{R_n^2}{5n\lambda}, \text{ and, so on.}$$

**Phase reversal zone plates.** It was suggested by Lord Rayleigh that, instead of blackening out the alternate zones, if the phases in these zones could be reversed, then four-fold increase in intensity would be obtained at the focus of the zone plate. This suggestion was put to practice by prof R. W. Wood (Vide Physical Optics by Wood. Third Edition pp 39-40)

31. **Diffraction at a Straight Edge.** For simplicity let us suppose that the source is linear (a straight filament, for example) so that the the wave-front round the source is cylindrical. Let a portion of this wave-front be intercepted by a semi-infinite opaque obstacle with a straight edge placed parallel to the linear source. Let BCD (Fig. v-6) be the section of the wave-front radiated from the source S by the plane of the paper. Let ROT be the section of the screen placed in the path of the beam and let O be the edge of the so-called geometrical shadow.

To find the illumination at the point R outside the geometrical shadow, join SR cutting the wave-front at P. Then P is the pole with respect to the point R. According to Fresnel-Huygen principle, the illumination at R is due to the superposed effect of disturbances coming from all the half-zones lying above the point P and the number of half-zones lying between the points P and C. As the point R moves towards O, the pole P also

moves with it and, one after another, the number of half-zones between P and C are intercepted by the screen, but the portion above P remains the same. Hence there will be general illumination at all points above O due to one half of the complete set of zones; superposed on this general illumination there would be another illumination whose intensity would fluctuate periodically

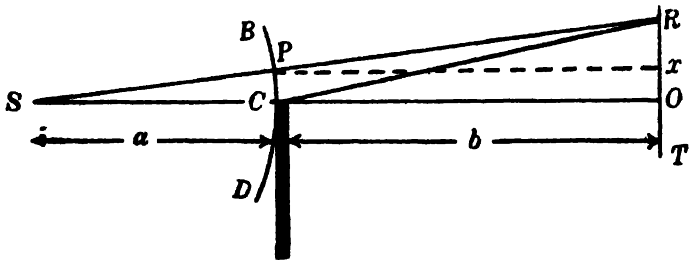



Fig. V-6

on the screen. This is because *the effect of odd number of exposed zones is much greater than that of even number of zones as* already explained (see P. 150). Hence on the screen above the geometrical shadow, there would be produced a number of fringes of alternately maximum and minimum intensity, the minimum intensity being equal to that of the general illumination due to all the half-zones lying above P.

If the point R lies inside the geometrical shadow as at T, its pole P′ would be intercepted by the screen, so that the one half of the total number of zones lying below P′ are completely cut out. Also some of the half-zones lying between the pole P′ and the edge C of the obstacle are intercepted, so that the total number of exposed zones are less than half of the set of zones. Now *if a number of zones nearest the pole are intercepted, the effect of the remaining exposed zones is equal to half of the first exposed zone that is nearest the pole. Further the effect of a zone diminishes as its distance from the pole increases.* Hence as the point T moves down into the geometrical shadow one after another of the zones lying above P are intercepted by the screen, so that the illumination at T gradually diminishes till it fades out into complete darkness.

Thus, the nature of illumination on the screen is as shown in Fig. 6, Plate VI. Below the edge of the geometrical shadow the illumination gradually fades away into complete darkness. Above the geometrical shadow, the illumination breaks up into discontinuities so as to produce a system of fringes which are alternately of maximum and minimum intensity, the fringes being gradually closer and closer till general illumination is obtained.

**Calculation of width of fringes.** Let the distances SC and CO be equal to $a$ and $b$ respectively and let the distance OR be equal to $x_s$. From what has been said above, the intensity at R is maximum or minimum according as the number of half-zones between P and C is odd or even. Hence, for maximum intensity

$$CR-PR=(2s+1)\frac{\lambda}{2} \qquad [s=0, 1, 2, 3, 4, \ldots]$$

and, for minimum intensity

$$CR-PR=2s\frac{\lambda}{2} \qquad [s=0, 1, 2, 3, 4, \ldots]$$

Now from geometry of the figure

$$CR=\sqrt{b^2+x_s^2}=b\left(1+\frac{x_s^2}{b^2}\right)^{\frac{1}{2}}$$

or $$CR=b+\frac{x_s^2}{2b}, \text{ neglecting higher order terms.}$$

Again, $$RS=\sqrt{(a+b)^2+x_s^2}=(a+b)\left[1+\frac{x_s^2}{(a+b)^2}\right]^{\frac{1}{2}}$$

or, $$RS=(a+b)+\frac{x_s^2}{2(a+b)}, \text{ neglecting higher order terms.}$$

Hence $$PR=RS-PS=b+\frac{x_s^2}{2(a+b)}$$

so that $$CR-PR=\left(b+\frac{1}{2}\cdot\frac{x_s^2}{b}\right)-\left(b+\frac{1}{2}\cdot\frac{x_s^2}{a+b}\right)$$

or, $$CR-PR=\frac{ax_s^2}{2b(a+b)}.$$

Hence, for maximum intensity, we have

$$\frac{ax_s^2}{2b(a+b)}=(2s+1)\frac{\lambda}{2}$$

or, $$(x_s)_{max.}=\sqrt{\frac{b(a+b)(2s+1)\lambda}{a}} \quad \dots \quad (31\cdot1)$$

For minimum intensity, we have

$$\frac{ax_s^2}{2b(a+b)}=2s\frac{\lambda}{2}$$

or, $$(x_s)_{min.}=\sqrt{\frac{b(a+b)2s\lambda}{a}} \quad \dots \quad (31\cdot2)$$

The width of the $s$th bright fringe is the distance between the centre of the $(s+1)$th dark fringe to the center of the $s$th dark fringe. Hence, by Eqn. (31-2)

$$x_{s+1}-x_s=\sqrt{\frac{b(a+b)2\lambda}{a}}\left[\sqrt{s+1}-\sqrt{s}\right]$$

Hence the width of a fringe depends on the order of the fringe, so that, the fringes are not all of the same width. The fringes get closer together as one moves away from the edge of the geometrical shadow. This unequal spacing of the fringes together with their poor contrast distinguishes the diffraction fringes from the interference fringes. Since the width of a fringe also depends on the wave-length, a system of rainbow coloured fringes would be obtained with white light.

**32. Diffraction at a Narrow Obstacle.** Let ABC (Fig. V-7) be the trace of the cylindrical wave-front by the plane of the paper originating from a linear source S. AB is the section of a narrow obstacle placed parallel to the source. Join SA and SB and produce them to meet the screen at N and M respectively, so that N and M are the boudaries of the geometrical shadow.

At a point *R* outside the geometrical shadow the illumination is due to the portion of the wave-front lying above the point A, the portion of the wave front lying below the point B having

very little influence, if the width of the obstacle is large compared to the wave-length of light. This would produce a system of diffraction fringes outside the edge N of the geometrical shadow.

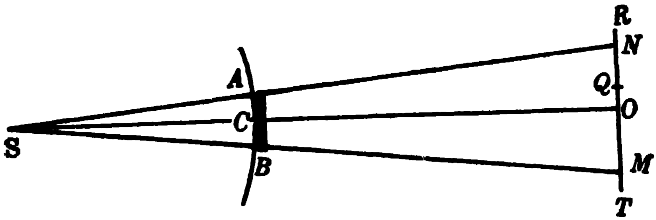



Fig. V-7

Similarly, there would be another system of diffraction fringes below the edge M formed by the portion of the wave-front lying below the edge B of the obstacle.

Any point Q inside the geometrical shadow will, however, be illuminated by light coming from the portions of the wave-fronts lying on either side of the obstacle. Now the effect of all the exposed zones is equal to half that of the first exposed zone which is nearest the pole. Hence, so far as the illumination inside the geometrical shadow is concerned, the portions of wave-fronts above A and below *B* may be replaced by two imaginary luminous sources each emitting radiation of amplitude equal to half that of the first exposed zone, on either side of the obstacle. These two luminous sources would, of course, be in the same phase (being derived from the same wave-front). They would, therefore, produce a system of interference fringes within the geometrical shadow. This was proved by Dr. Young, for he showed that the interference fringes inside the geometrical shadow disappeared if one of the exposed portions of the wave-fronts on either side of the obstacle were intercepted by a screen. If the width of the obstacle is very large, there would be two diffraction patterns similar to two straight edges as described in Art. 31. When the width of the obstacle is very small, the interference fringes become very wide.

**33. Diffraction through a Narrow Rectangular Aperture.** In Fig. V-8 let S be the linear source, AB the aperture and, QT the screen. Let the points T and N be the borders of the geometrical shadow.

To find the illumination at the point Q outside the geometrical shadow, join SQ, cutting the wave-front at P, so that P is the pole with respect to the point Q on the screen. Divide the exposed wave-front into half-period elements. Evidently the illumination

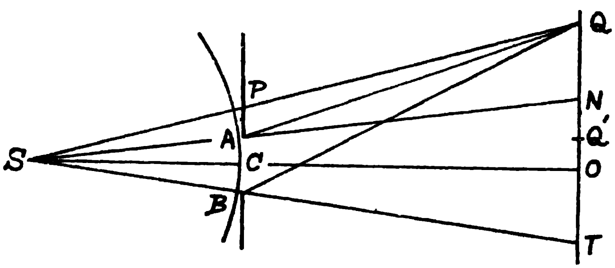



Fig. V-8

at Q will be maximum or minimum according as the number of exposed half-zones is odd or even. This argument also holds for the portion of the screen below T.

The pattern of fringes inside the geometrical shadow will depend on the position of the screen relative to the aperture. The area of a zone increases as the distance of the screen from the wave-front increases. Hence, if the distance of the screen from the source is such that, with respect to the point Q′, AB contains only a single half-element, then T and N would only be borders of the first fringe. As the screen is moved towards AB, the number of half-elements inside AB with respect to the points T and N would increase. Accordingly, the intensity of illumination at a point Q′ inside the geometrical shadow would be determined by the joint effect of the number of half-elements lying on either side of the pole with respect to Q′. This would generally produce a system of fringes inside the geometrical shadow.

**34. Diffraction at a Circular Aperture.** We shall consider here only a particular case, namely, the illumination at a point on the axial line passing through the centre of the circular aperture.

Let S (Fig. V-9) be apoint-source of light and AB the circular aperture. Let SPQ be the axial line. Then the centre P of the

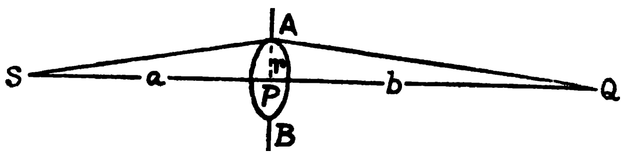



Fig. V-9

aperture is the pole with respect to the point Q. Divide the wave-front into half-period elements. Then the intensity of illumination at Q will be maximum or minimum according as the aperture AB contains an odd or even number of half-period elements. The positions of points of maximum or minimum intensity can be calculated as follows :

Let the distance SP$=a$, PQ$=b$, and the radius of the aperture $r$.

Then $\qquad QA^2=b^2+r^2$

or $\qquad QA=b\left(1+\frac{r^2}{b^2}\right)^{\frac{1}{2}}=b\left(1+\frac{r^2}{2b^2}\right)$

neglecting higher order terms :

or $\qquad QA=b+\frac{r^2}{2b}$

Similarly, $\qquad SA=a+\frac{1}{2}\frac{r^2}{a}$

Hence $\quad QA+SA=(a+b)+\frac{r^2}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$

The path difference between the central and the marginal rays is

$$\delta=(QA+SA)-(SP+PQ)=\frac{r^2}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$$

so that $\qquad r^2=\frac{2ab\delta}{a+b}.$

The area of the circular aperture is

$$\pi r^2 = \frac{2\pi ab\delta}{a+b}.$$

Putting $\delta = s\lambda/2$, where $s$ may be an odd or even number, we get

$$\pi r^2 = \frac{\pi s\, ab\lambda}{a+b}$$

or
$$r^2 = \frac{s\, ab\lambda}{a+b}$$

so that
$$b = \frac{ar^2}{s\, a\lambda - r^2}$$

Since $a$, $r$, and $\lambda$ are constants, the value of $b$ will depend on the value of s. As $s$ passes through successive integral values which are alternately odd or even, $b$ changes correspondingly and therewith the intensity also passes through a series of maximum and minimum values. Hence, if the screen be moved towards the aperture, the central spot on the screen will be alternately of maximum and minimum intensity. Again since the value of $b$ depends on the wave-length $\lambda$, the points of maximum and minimum intensity of light of different wave-lengths would be situated at different distances from the aperture AB.

**35. The Plane Transmission Grating** : The plane diffraction grating is an optically plane glass plate with a large number of parallel and equidistant lines etched on one of its faces (Fig. V-10). When a beam of light falls on the surface of the grating the portions of light incident on the ruled spaces are scattered out in all directions while the rays falling on the transparent spaces between the rulings are freely transmitted. A grating may thus be regarded as made up of a large number of narrow and equidistant parallel slits arranged on a plane surface.

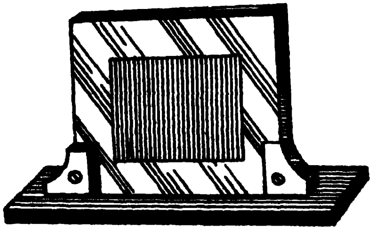


Fig. V-10

The width of the rulings or transparent spaces is very small compared to their lengths.

If $a$ be the width of a ruling and $b$ that of a transparent space, then $(a+b)$ is called the *grating space.*

**The optical behaviour of a grating.** When a plane wave-front is incident on the grating, the portions of wave-front incident on the transparent spaces give rise to Huygens secondary wavelets that transmit rays in all directions. These rays will be referred to as diffracted ray. If a converging lens is placed in the path of the diffracted rays, all rays from the different secondary sources which are diffracted at the same angle with the normal to the grating surface are brought to a

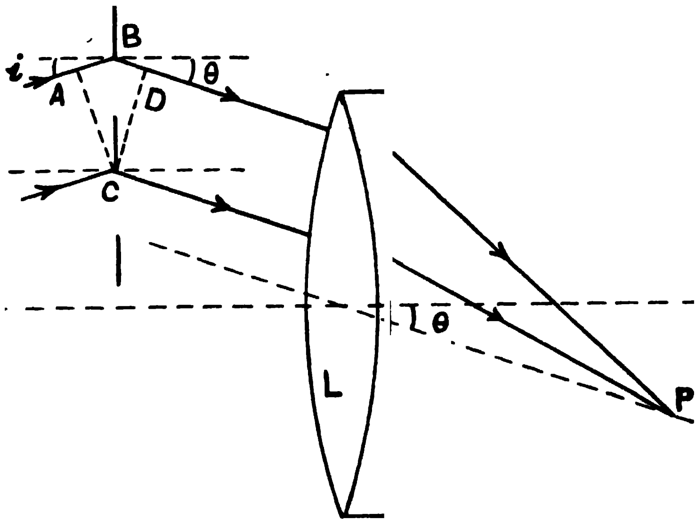



Fig. V-11

focus at P (Fig. V-11) on the secondary axis of the lens that is parallel to these diffracted rays. If the rays meeting at the point P agree in phase, then the intensity at the point P would be maximum. If they are completely out of phase (phase angle 180°) then the intensity at P would be minimum.

**Conditions of maxima and minima.** To take a general case, let a beam of parallel rays be incident on the grating at an angle $i$. Now the path-difference between two rays

diffracted at angle $\theta$ from two similarly situated points B and C in two consecutive slits as in Fig. V-11, is $l=AB+BD$, where A and D are the feet of the perpendiculars dropped from C on the rays AB and BD respectively. Hence

$$l=AB+BD=(a+b)(\sin i+\sin\theta),$$

$(a+b)$ being the grating space. The phase-difference of these two rays is

$$\delta=\frac{2\pi l}{\lambda}=\frac{2\pi}{\lambda}(a+b)\ (\sin i+\sin\theta)$$

It is evident that the path-difference between the ray diffracted at B and that diffracted at the same angle from any similarly situated point in any other slit would be an integral multiple of $l$. Hence all such rays diffracted at the same angle $\theta$ from similarly situated points on all the slits would agree in phase. Again since the path between the diffracted wave-front CD and the secondary focus P of the lens is the same for all rays, $\delta$ is also the phase-difference between the incident wave-front AC and the focus P of the lens.

The intensity at the point P would be maximum, if

$$(a+b)(\sin i+\sin\theta)=2s\frac{\lambda}{2}, \quad \ldots \quad (35{\cdot}1)$$

where $[s=0, 1, 2, 3, 4,\ldots]$

and it would be minimum if

$$(a+b)\ (\sin i+\sin\theta)=(2s+1)\ \frac{\lambda}{2} \quad \ldots \quad (35{\cdot}2)$$

where $[s=0,1,2,3,4,\ldots]$

In Eqns. (35·1) and (35·2) the value of $\theta$ is determined by $s$, all other quantities involved in the equations being constants. The point of minimum intensity would thus be situated at different angles, thus giving rise to a system of bright lines parallel to the length of the rulings. The line corresponding to a particular value of $s$ is called the **sth order spectrum.** Thus the spectrum corresponding to $s=1$, 2, 3, etc. are called the first, second, third, etc. order spectrum, the zeroeth order spectrum being the undeviated image corresponding to $\theta=0$.

In case of normal incidence $i=0$, and the conditions of maxima and minima are respectively

$$(a+b)\sin\theta=s\lambda$$

and $$(a+b)\sin\theta=(2s+1)\frac{\lambda}{2}$$

The total number of spectra formed by a given grating depends on the number of lines in the grating. To take a numerical example, let the grating have 14000 lines to the inch or 5512 lines to the centimetre, so that the grating space is

$$a+b=\frac{1}{5512}=1{\cdot}8\times10^{-4}\text{ cm.}$$

If the incident light is of wave-length $5893\times10^{-8}$ cm. the angle of diffraction $\theta_1$ for the first order spectrum $(s=1)$ for normal incidence $(i=0)$ is given by

$$1{\cdot}8\times10^{-4}\sin\theta_1=5893\times10^{-8}$$

$$\sin\theta_1=0{\cdot}327$$

$$\theta_1=19^\circ\text{ nearly.}$$

The diffraction angle $\theta_2$ for the second order spectrum $(s=2)$ is given by $$1{\cdot}8\times10^{-4}\sin\theta_2=2\times5893\times10^{-8}$$

This gives $$\sin\theta_2=0{\cdot}655$$

$$\theta_2=41^\circ$$

The diffraction angle $\theta_3$ for the third order spectrum $(s=3)$ is given by

$$1{\cdot}8\times10^{-4}\sin\theta_3=3\times5893\times10^{-8}$$

This gives $$\sin\theta_3=0{\cdot}982$$

$$\theta_3=79^\circ$$

The diffraction angle $\theta_4$ for the fourth order spectrum $(s=4)$ is given by

$$1{\cdot}8\times10^{-4}\sin\theta_4=4\times5893\times10^{-8}$$

This gives $$\sin\theta_4=1{\cdot}309$$

and no real value of $\theta_4$ can be obtained.

The above calculation gives only three spectra between 0° and 90°. The spectra, therefore, consist of sharp lines separated by very wide dark intervals. It can be similarly shown that the total number of spectra formed by a grating diminishes as the number of lines per cm. of the grating increases.

**Minimum deviation of a grating.** For the $s$th order spectrum, we have

$$(a+b)(\sin i+\sin\theta_s)=s\lambda \qquad \dots \quad (35\cdot3)$$

The deviation of rays diffracted in the direction $\theta_s$ is

$$D=i+\theta_s$$

To find the minimum value of $D$, we equate the differential of $D$ to zero, thus getting

$$dD=di+d\theta_s=0$$

$$di=-d\theta_s \qquad \dots \quad (35\cdot4)$$

By differentiating Eqn. (35·1), we get

$$(a+b)(\cos i\,di+\cos\theta_s\,d\theta_s)=0$$

$$\cos i\,di+\cos\theta_s.\,d\theta_s=0.$$

Substituting for $d\theta_s$ from Eqn. (35·4) we get

$$\cos\theta_s=\cos i$$

or

$$i=\theta_s$$

so that the minimum value of deviation is

$$D_m=2i.$$

Thus, the deviation of the diffracted ray is a minimum when the angle of diffraction is equal to the angle of incidence. Substituting the minimum value of the angle of diffraction in Eqn. (35·4) we get

$$2(a+b)\sin i=s\lambda$$

$$2(a+b)\sin\tfrac{1}{2}D_m=s\lambda$$

**Alternative method of calculation.** To determine the conditions of maxima and minima, we have calculated the path-difference between two parallel rays diffracted at the same angle $\theta$ from two similarly situated points with respect to two consecutive

slits, and proved that the path-difference between two such rays in the $s$th order spectrum is

$$l_1=(a+b)\ (\sin i+\sin \theta_s)=s\lambda$$

The path-difference between corresponding rays in the first and third slits is

$$l_2=2(a+b)\ (\sin i+\sin \theta_s)=2s\lambda$$

If $N$ is the total number of slits, the maximum path-difference between the corresponding points in the first and the last slit is

$$l_N=N(a+b)\ (\sin i+\sin \theta_s)=Ns\lambda \quad \ldots \quad (35{\cdot}6)$$

We can write Eqn. (35·6) in the form $l_N=2\left(\frac{N}{2}s\lambda\right)$

Maximum path difference = double the difference between the first and the $\left(\frac{N}{2}+1\right)$th slits.

This means that the same condition of maxima and minima can be obtained if we suppose the grating to be divided into two equal halves* and consider the interference between similarly situated points in the first slit of the first half and in the $\left(\frac{N}{2}+1\right)$th slit or the first slit in the second half, the second slit of the first half and the second slit of the second half, the third slit of the first half and third slit of the second half and, so on.

**36. Secondary Maxima and Minima.** The maxima satisfying the conditions (Eqn. 35·1) are called the *primary maxima.* In between two consecutive primary maxima there are situated other maxima and minima which are too faint to be visible particularly when the number of slits is large. The evidence of the existence of the *secondary maxima and minima* can be obtained in the following way :

*If the number $N$ of the slits be odd, then the grating may be divided into two groups of $(N-1)/2$ slits. The contribution by the remaining slit may be neglected when $N$ is large.

For the sake of simplicity we shall suppose that the incidence is normal, so that the angle of diffraction for the $s$th maximum as given by Eqn. (35·6) is

$$\mathcal{N}(a+b)\sin\theta_s=\mathcal{N}s\lambda \quad \dots \quad (36\cdot1)$$

and that for the $(s+1)$th maximum is given by

$$\mathcal{N}(a+b)\sin\theta_{s+1}=\mathcal{N}(s+1)\lambda=\mathcal{N}s\lambda+\mathcal{N}\lambda.. \quad \dots(36\cdot2)$$

where $\mathcal{N}$ is the total number of rulings or transparent spaces.

Let us now confine our attention to the region of space between the angles $\theta_s$ and $\theta_{s+1}$ given by the above two equations. To move from $\theta s$ to $\theta_{s+1}$ the path difference must change by $\mathcal{N}\lambda$. Let $\theta s_1$, $\theta s_2$, $\theta s_3$, etc. be some intermediate values of $\theta$ between $\theta s$ and $\theta s_{+1}$ which correspond to increments of the path-difference $\mathcal{N}s\lambda$ by $\lambda/2$, $2\lambda/2$, $3\lambda/2$......$2\mathcal{N}\lambda/2$, so that

$$\mathcal{N}(a+b)\sin\theta_{s1}=\mathcal{N}s\lambda+\lambda/2 \quad \dots \quad (36\cdot3)$$

$$\mathcal{N}(a+b)\sin\theta_{s2}=\mathcal{N}s\lambda+2\lambda/2 \quad \dots \quad (36\cdot4)$$

$$\mathcal{N}(a+b)\sin\theta_{s3}=\mathcal{N}s\lambda+3\lambda/2 \quad \dots \quad (36\cdot5)$$

$$\dots \quad \dots \quad \dots$$

$$\mathcal{N}(a+b)\sin\theta_{s+1}=\mathcal{N}s\lambda+2\mathcal{N}\frac{\lambda}{2}$$

Let us now consider these cases of small increments of path separately.

**Case 1.** $\mathcal{N}(a+b)sin\theta_{s1}=\mathcal{N}s\lambda+\lambda/2$. This can also be written as

$$\mathcal{N}(a+b)\sin\theta_{s1}=2\left(\frac{\mathcal{N}}{2}s\lambda+\lambda/4\right).$$

In this case the path-difference between two rays diffracted at angle $\theta_{s1}$ from the corresponding slits in the two halves AB and BC (Fig. v-12) of the grating is $\lambda/4$. If the equation of vibration from the first slit in the first half is

$$y_1=A\cos\omega t,$$


Fig. 12

that from the $\left(\frac{\mathcal{N}}{2}+1\right)$th slit would be

$$y_2=A\cos\left[\omega t+\frac{2\pi}{\lambda}\left(\frac{\mathcal{N}}{2}s\,\lambda+\frac{\lambda}{4}\right)\right]$$

or $$y_2=-A\sin\omega t.$$

Hence the resultant vibration would be

$$y' = y_1 + y_2 = A\ (\cos \omega t - \sin \omega t).$$

Total displacement due to $N/2$ pairs of slits is

$$y = \frac{NA}{2}(\cos \omega t - \sin \omega t)$$

so that, the resultant amplitude in the direction $\theta_{s1}$ is

$$\alpha = \frac{NA}{2}$$

**Case 2.** $N(a+b)\sin\ \theta_{s2} = Ns\lambda + \frac{2\lambda}{2} = 2\left(\frac{N}{2}s\lambda + \frac{\lambda}{2}\right)$

Hence supposing the grating to be divided into two equal parts the displacements due to the first and the $\left(\frac{N}{2}+1\right)$th slits are respectively

$$y_1 = A \cos \omega t$$

and

$$y_2 = A \cos\left[\omega t + \frac{2\pi}{\lambda}\left(\frac{N}{2}s\lambda + \frac{\lambda}{2}\right)\right]$$

$$= -A \cos \omega t$$

so that, the resultant displacement is

$$y' = y_1 + y_2 = 0.$$

Hence the intensity in the direction $\theta_{s2}$ is zero.

**Case 3** Let $N(a+b) \sin\ \theta_{s3} = Ns\lambda + \frac{3\lambda}{2}$.

To simplify the calculation, this equation may be written to the form

$$N(a+b)\ \sin\ \theta_{s3} = 3\left(\frac{N}{3}s\lambda + \frac{\lambda}{2}\right)$$

Accordingly, if the grating be supposed to be divided into three equal parts, AB BC, and CD (Fig. V-13), the path-difference bet-


Fig. V-13

ween two corresponding slits in AB and BC would be $\lambda/2$ in the particular direction $\theta_{s3}$, so that the disturbances from these two elements mutually destroy each other as in case 2. Hence they contribute nothing to the illumination in the direction $\theta_{s3}$. Only contribution is, therefore, due to the corresponding slit in the remaining element CD with $N/3$ slits, and with the path

difference $\lambda/2$ between extreme points C and D. The case is, therefore, similar to case I with the difference that for $\mathcal{N}$ we have to substitute $\mathcal{N}/3$, Hence the resultant amplitude is $\frac{1}{3}$ of case I.

**Case 4.** Let $\mathcal{N}(a+b)\ \theta_{s_4} = \mathcal{N}s\lambda + \frac{4\lambda}{2} = 4\left(\frac{\mathcal{N}s\lambda}{4} + \frac{\lambda}{2}\right)$

If we divide the grating into 4 equal parts, AB, BC, CD, and DE (Fig. V-14) then AB and BC as also CD and DE would

Fig. V-14

neutralise the effect of one another, since the path-difference between corresponding slits would be $\lambda/2$. Hence in the direction $\theta_{s_4}$ the intensity would be zero.

**Case 5.** $\mathcal{N}(a+b)\ \sin\theta_{s_5} = \mathcal{N}s\lambda + 5\lambda/2 = 5\left(\frac{\mathcal{N}s\lambda}{5} + \frac{\lambda}{2}\right)$

In this case we can divide the grating into five equal parts AB,BC,CD,DE,and EF. The elements (AB,BC) and (CD,DE) would neutralise the effect of one another, so that, the only effective element is EF with $\mathcal{N}/5$ slits. This is similar to case I, the difference being that here we have got $\mathcal{N}/5$ slits instead of $\mathcal{N}$. Hence the amplitude is only one-fifth of case I.

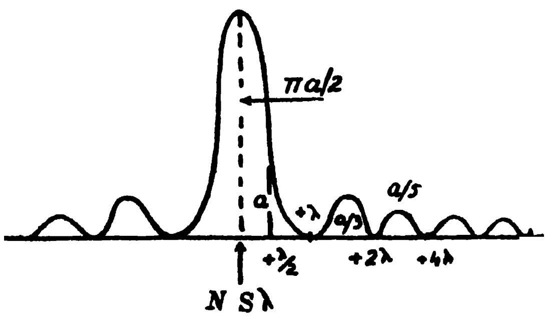


Fig. V-15

The distribution of intensity on two sides of the $s$th primary spectrum would be as shown in Figure V-15.

**37. Width of the Primary Spectrum.** From what has been said above (Fig. V-15), half the angular width of the primary spectrum of order $s$ is the distance between the primary maximum and the next secondary minimum, that is

$$\triangle\theta_s=\theta_{s_2}-\theta_s.$$

Assuming normal incidence

$$\begin{aligned}N(a+b)\sin\theta_{s_2}&=N(a+b)\sin(\theta_s+\triangle\theta_s)\\&=N(a+b)(\sin\theta_s\cos\triangle\theta_s+\cos\theta_s\sin\triangle\theta_s)\\&=N(a+b)(\sin\theta_s+\triangle\theta_s\cos\theta_s)\end{aligned}$$

From case 2, $N(a+b)(\sin\theta_s+\triangle\theta_s\cos\theta_s)=Ns\lambda+\lambda$ ...(37·1)

Also, from Eqn. (36.1) $N(a+b)\sin\theta_s=Ns\lambda$ ... (37·2)

Subtracting Eqn. (37.2) from Eqn. (37.1) we get

$$N(a+b)\,\triangle\theta_s\cos\theta_s=\lambda$$

$$\triangle\theta_s=\frac{\lambda}{N(a+b)}\,\frac{}{\cos\theta_s} \quad \text{... (37 3)}$$

Hence the spectral lines would be sharper as the total number of lines $N$ in the grating is increased.

**38. Dispersion produced by a Grating.** For normal incidence the angle of diffraction $\theta_s$ for the $s$th order spectrum is given by

$$(a+b)\sin\theta_s=s\lambda$$

Since $\theta_s$ depends on $\lambda$, the angle of diffraction for different wavelengths would be different. Hence if a heterogeneous beam of light be incident on the grating the diffracted ray would be dispersed into differently coloured constituents. For the same order spectrum the angle of diffraction would increase with the wavelength.

**The normal spectrum.** In the case of prismatic spectra dispersion depends on the nature of material of the prism. Further, in the case of prismatic spectra some amount of anomaly is observed particularly near the region of absorption where the order of colours in the spectrum is reversed. But with a grating the angle of diffraction increases with the

wave-length. It is for this reason that grating spectra are called the normal spectra.

**Angle of dispersion.** Let us now calculate the angle of dispersion between two colours differing in wave-length by $\delta\lambda$ For the $s$th order spectrum, we have

$$(a+b)(\sin i+\sin \theta_s)=s\lambda \qquad \ldots \quad (38.1)$$

Let $\theta_s$ change to $(\theta_s+\delta\theta_s)$ when the wave-length $\lambda$ changes to $\lambda+\delta\lambda$.

Then $(a+b)[\sin i+\sin(\theta_s+\delta\theta_s)]=s(\lambda+\delta\lambda)$

or $\quad (a+b)[\sin i+(\sin \theta_s+\cos \theta_s.\ \delta\theta_s)]=s(\lambda+\delta\lambda). \quad \ldots(38\ 2)$

Subtracting Eqn. (38.1) from Eqn. (38.2) we get

$$(a+b)\cos \theta_s \delta\theta_s=s.\delta\lambda$$

so that, the angle of dispersion between wave-lengths $\lambda$ and $\lambda+\delta\lambda$ is

$$\delta\theta_s=\frac{s.\delta\lambda}{(a+b)\cos\theta_s} \qquad \ldots \quad (38.3)$$

It is to be noted that $\delta\theta_s$ in Eqn. (38·3) is the angular separation between the points of maximum intensity for the wave-lengths $\lambda$ and $\lambda+\delta\lambda$ in the $s$th order spectrum. Hence dispersion increases with the order of the spectrum. It also increases if the grating space diminishes. Hence with higher order spectrum there may be overlapping of spectra of two consecutive orders. For lower order spectra for which $\theta_s$ is small, so that, $\cos\theta_s$ is nearly equal to unity, $\delta\theta_s$ is proportional to $\delta\lambda$; in other words, dispersion is proportional to difference in wave-length. This is in marked contrast with prismatic spectra.

**39. Chromatic Resolving Power of a Grating**—We have seen that when two radiations of wave-lengths $\lambda$ and $\lambda+\delta\lambda$ fall on a grating their primary maxima in the $s$th order are separated by an angular distance AB (Fig. V-16) given by

$$AB=\delta\theta_s=\frac{s.\delta\lambda}{(a+b)\cos\theta_s}.$$

The semi-angular widths $C_1D_1$ and $C_2D_2$ of the primary spectrum (Fig. V-16) for each of these wave-lengths has been found to be (Eqn. 37.3)

$$C_1D_1=C_2D_2=\Delta\theta_s=\frac{\lambda}{N(a+b)\cos\theta_s}.$$

It is evident from Fig. V-16 that the two primary maxima of wave-lengths $\lambda$ and $\lambda+\delta\lambda$ would evidently appear as separate if $\delta\theta_s$ is greater than $2\Delta\theta_s$.

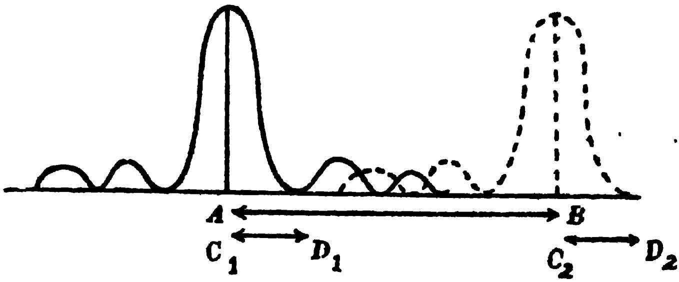



Fig. V-16

By diminishing $\delta\lambda$, $\delta\theta_s$ is also diminished and the two primary maxima corresponding to wave-lengths $\lambda$ and $\lambda+\delta\lambda$ approach one another till they overlap and can hardly be distinguished as two separate lines. Regarding the limit to the closeness of approach of the two primary maxima which can yet be recognised as separate by the normal eye, Lord Rayleigh has pointed out that this limiting value of $\delta\lambda$ (and consequently that of $\delta\theta_s$) must be such that the primary maximum of wave-length $\lambda$ is superposed on the first minimum of wave-length $\lambda+\delta\lambda$. Under this condition the two lines appear as a single broad line with slight decrease of intensity at the centre (Fig. V-17). If the two lines are equally intense, then the intensity at the centre of the superposed spectra is $8/\pi^2$ (or 0·81) time their maximum intensity.

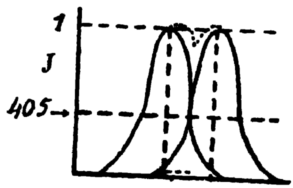



Fig. V-17

Under this circumstance limiting condition of overlapping is $\Delta\theta s=\delta\theta_s$. Hence the smallest difference in wave-lengths of two lines which can be recognised as just separate or 'resolved' (as this limiting condition of separation is technically called) is given by

$$\frac{s.\delta\lambda}{(a+b)\cos\theta_s}=\frac{\lambda}{N(a+b)\cos\theta_s}$$

$$\delta\lambda=\frac{\lambda}{Ns}.$$

Coventionally the ratio of $\lambda$ to the limiting value of $\delta\lambda$ is called the resolving power of the grating or,

$$\textit{Resolving power} = \frac{\lambda}{\delta\lambda} = Ns. \quad .. \quad (61.1)$$

Thus the resolving power does not depend on the grating space $(a+b)$, but only on the total number of lines $(N)$ and on the order of the spectrum. Evidently to take the fullest advantage of the resolving power of a grating, the whole of the ruled surface of the grating must be illuminated by the light under examination.

The yellow sodium line seen in the spectrometer is really a combination of two lines called the $D_1$ and $D_1$ lines having wave-lengths $5896 \times 10^{-8}$cm. and $5890 \times 10^{-8}$ cm. Hence their difference of wave-length is $6 \times 10^{-8}$cm. In order that these two lines may appear as just separate, the resolving power of the grating must be

$$\frac{\lambda}{\delta\lambda} = \frac{5890 \times 10^{-8}}{6 \times 10^{-8}} = 1000 \text{ nearly,}$$

or, $$Ns = 1000.$$

Hence, if the lines are to appear as separate in the first order spectrum $(s=1)$, the grating must have at least 1000 lines. If they are to appear as resolved in the second order spectrum $(s=2)$ the grating must have at least 500 lines.

**62. Measurement of Wave-length of light with Grating.** The diffraction grating affords one of the accurest methods of measurement of wave-length of light. This is done with the help of a spectrometer. For this purpose the spectrometer is levelled and set for parallel rays in the usual way (See Part 1, P. 393). The grating is placed on the prism table upon a stand with its ruled surface facing the telescope and containing the axis of rotation of the prism table. For convenience of measurement the light from the collimator is made to fall on the grating surface normally.

To set the instrument for normal incidence, the grating is removed from the table, the telescope is turned towards the collimator and the image of the slit is received on the point of intersection of the cross-wires and the reading of the vernier attached to the telescope is taken. The telescope is then turned

through 90° from this position. The grating is then placed on the prism table with its face perpendicular to the line joining the two screws $S_2$ and $S_3$ (Fig. V-18) and the ruled face containing the axis of rotation. The table with the grating is then slowly rotated till the image of the slit formed by reflection from the grating face coincides with the point of intersection of the cross-wires when the reading of the prism tables is taken. On turning the prism table through 45° from this position, the surface of the grating becomes perpendicular to the incident rays.

The next adjustment required is to set the rulings parallel to the slit. For this purpose the slit of the collimator is made vertical. The telescope is rotated to receive the first order spectrum on the cross-wire. On turning the levelling screw $S_1$ of the prism table the inclination of the rulings to the vertical is changed and the spectrum appears to move up or down in the field of view of the telescope and at the same time its definition changes. The levelling screw $S_1$ is adjusted till the first order spectrum becomes as sharply defined as possible.

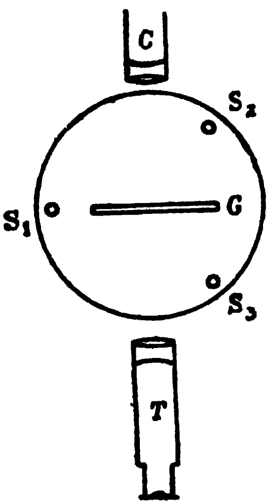


Fig. V-18

To determine the wave-length of light, the slit of the collimator is illuminated by the light under examination. The telescope is then turned to receive the first order image on either side of the central image and the two readings are taken. Half the angle between the two positions of the telescope gives the angle of diffraction $\theta_1$ of the first order spectrum. Knowing the value of $\theta_1$, the wave-length is calculated from the relation

$$(a+b)\sin\theta_1=\lambda$$

or

$$\sin\theta_1=\frac{\lambda}{a+b}.$$

This method is repeated for other order spectra. The grating space $(a+b)$ is given by the maker of the grating. It can also be determined by performing the experiment with a known wave-length of light.

**63. Determination of Resolving Power of a Grating.** To determine the resolving power of a grating experimentally, set up the grating in the manner described above. Illuminate the slit of the collimator by sodium light. Next place an adjustable slit between the collimator and the grating and slowly reduce the width of the slit so as to cut down the effective width of the grating till the two sodium lines just run together. Then measure the width of the slit. Let this width be $b$. If $N'$ be the number of rulings per cm, the resolving power of effective aperture of the grating is

$$R' = bsN'$$

where $s$ is the order number of the spectrum, or $N's = R'/b$.

If $a$ be the whole width of the ruled surface $aN's = aR'/b$.

But $aN' = N =$ the total number of lines in the grating.

Hence the resolving power of the grating is

$$R = Ns = aR'/b.$$

# CHAPTER VI

## DOUBLE REFRACTION AND POLARISATION OF LIGHT

**42. The Crystalline Medium.** Hitherto we have been chiefly concerned with propagation of light in a homogeneous and isotropic medium. We propose to study in this chapter the modes of propagation of light in an anisotropic medium such as the medium of matter in the crystalline state. On account of geometrical symmetries of arrangement of constituent particles of crystals, they are found to possess directional properties. Thus, physical properties such as elasticity, thermal expansion, electrical conductivity, and optical properties of a crystal are different in different directions. A crystalline medium is homogeneous but not isotropic; this means that the physical property of a crystal in a given direction is the same at all points, but at a given point they are different in different directions. We shall consider here only the optical properties of a crystalline medium.

According to the elastic-solid theory, velocity of light depends on elasticity of ether which is supposed to be modified by its association with matter. It is natural to suppose that in transmission of light through a crystalline medium the velocity of light and, therefore, the refractive index of the medium of the crystal will be different in different directions. In the beginning it will be useful to study some of the most important directional properties of certain types of crystal.

**Cleavage.** The most striking feature of a crystal is that it is bounded by perfectly plane faces which are inclined to one another. Most of the crystals (a notable exception is quartz) exhibit a marked tendency to break across definite planes which are always parallel to faces or possible faces of the crystals. Thus, if a knife edge is pressed against an edge of a crystal, it produces a clean break showing two new faces having a pearly or glassy lustre. The extreme ease with which

mica can be split into thin sheets is well-known. The operation described above is called cleaving and the plane faces produced by cleaving are called cleavage faces.

**Constancy of interfacial angles.** Crystals of the same substance are not always of identical shape due to accidental growth of some faces in preference to others. But in the multiplicity of forms of crystals of a single substance, there is one regularity—namely, the constancy of the angles between different faces. Thus, the ideal shape of a quartz crystal is

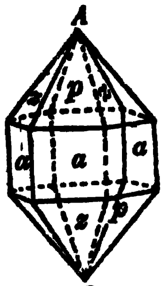



Fig. VI-1

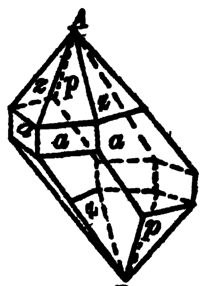



Fig. VI-2

shown in Fig. VI-1 and that of a deformed quartz crystal is shown in Fig. VI-2. The angle between the faces $(a,a)$, $(p,z)$ and $(p,a)$ are respectively 120°, 46° 16′, and 38′ 13°. This constancy of interfacial angles is maintained in both the types of quartz crystal shown in Figs. VI-1 and VI-2. The same interfacial angles would also be exhibited between the cleavage faces of the quartz crystal.

**The crystallographic axes.** The plane faces bounding a crystalline medium and its interfacial angles being of primary importance in determining the shape of a crystal, it is necessary to devise a method for specification of these bounding faces. For this purpose three edges (sometimes four) of the crystal* meeting at a point and not lying in the same plane are

*Theoretically any three axes may be chosen as crystallograppic axes. But from the study of the external form and symmetry of the crystal, certain set of axes suggests itself as most convenient.

chosen as the axes of reference. The axes thus chosen are called the *crystal axes* or *crystallographic axes.* The axes are different foı crystals of different substances. For each substance a plane ABC (Fig. VI-3) parallel to one of the bounding faces of the crystal is chosen which is inclined to all the crystal axes such that the intercepts OA=$a$, OB=$b$, OC=$c$ on the three axes OX, OY, and OZ are as nearly equal as possible. This face is called the *parametral plane.* The ratios $a/b$ $b/b$ and $c/b$ are called the *axial ratios.* The

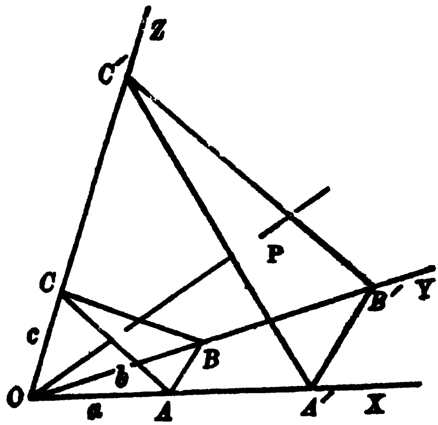



Fig. VI-3

inter-axial angles ($\alpha$, $\beta$, $\gamma$) and the axial ratios are collectively known as *crystal elements.*

**The law of rational indices.** Let A′B′C′ (Fig. VI-3) be a plane parallel to any other bounding face of the crystal intercepting the axes OX, OY, OZ at the points A′, B′, and C′, The experimental result is that the intercepts

$$OA' : OB' : OC' = Ha : Kb : Lc. \qquad \ldots \quad (42{\cdot}1)$$

where $H$, $K$, and $L$ are *whole numbers* and $a$,$b$,$c$, are the intercepts by the parametral plane on the three crystallographic axes. For planes which actually occur in crystals, $H$,$K$, and $L$ are usually small. In words, Eqn. (42·1) can be expressed as follows :

*The intercepts on the crystal axes by a plane parallel to any of the possible faces of the crystal can always be expressed as rational multiplies of the parameters.* This is known as the law of rational indices.

**The Miller indices.** Equation (42.1) can also be written as

$$OA' : OB' : OC = \frac{a}{KL} : \frac{b}{HL} : \frac{c}{HK}$$

Putting $KL=h$, $HL=k$ and $HK=l$, Eqn. (51·1) reduces to

$$OA' : OB' : OC' = \frac{a}{h} : \frac{b}{k} : \frac{c}{l} \qquad \ldots \quad (42{\cdot}2)$$

The lengths $a$, $b$, and $c$ are constants of a particular crystal, being intercepts on the crystallographic axes by the parametral plane. Hence possible faces of the crystal are defined by the values of $a/h$, $b/k$ and $c/l$. This is written as $(hkl)$. The quantities $h$, $k$, and $l$ are known as *Miller indices*. It is evident that for the parametral plane the Miller indices are (111). The negative value of the intercepts are represented by negative sign placed above the corresponding Miller index. Thus, the plane opposite the parametral plane is represented by $(\bar{1}\bar{1}\bar{1})$,

When a face of the crystal is parallel to one of the axes, the corresponding intercept on the axis is infinity, so that the corresponding Miller index is zero.

Eqn. (42.2) also admits of another interpretation. Let P be the foot of the perpendicular dropped from the origin on the plane A′ B′ C′. Let $\lambda$, $\mu$, and $\nu$ be the cosines of the angle POA′, POB′, and POC′ respectively. Then, from the right-angled triangles POA′, POB′, and POC′,

$$OP = OA'.\lambda = OB'.\mu = OC'.\nu$$

$$\therefore \quad \lambda : \mu : \nu = \frac{1}{OA'} : \frac{1}{OB'} : \frac{1}{OC'}$$

$$\text{or,} \quad \lambda : \mu : \nu = \frac{h}{a} : \frac{k}{b} : \frac{l}{c}$$

Thus the reciprocals of the Miller indices are proportional to the direction-cosines of the normal to the plane $(h\ k\ l)$

**Symmetry properties of a crystal.** A crystal is a geometrical figure possessing certain symmetry elements; this means that, by performing certain geometrical operations such as

rotation about a certain axis through a certain angle, or by reflection across a plane, or by combinations of rotation and reflection, the crystal can be brought into self-coincidence.

Observation shows that crystals can have only 2-fold, (or digonal), 3-fold (or trigonal), 4-fold (or tetragonal), and 6-fold (or hexagonal) axes of symmetry but never a 5-fold axis. By $n$-fold axis of symmetry we mean that by rotating the crystal through an angle $2\pi/n$ about the axis, the crystal is brought into self-coincidence. Any complicated structure of a crystal can be reduced to simple (or further irreducible) forms by cleavage. These simplest structures are called **unit structures** of which there are seven main systems. These are enumerated below :

(1) **The triclinic system.** It possesses no symmetry at all or at most only a centre of symmetry. It is referable to three

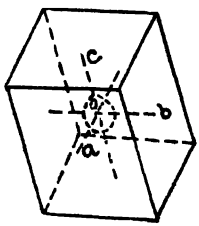



Fig. VI-4

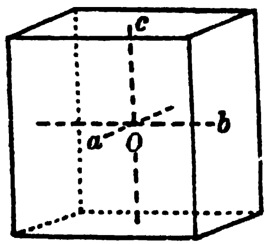



Fig. VI-5

unequal crystallographic axes* which are inclined at angles not equal to one another (Fig. VI-4).

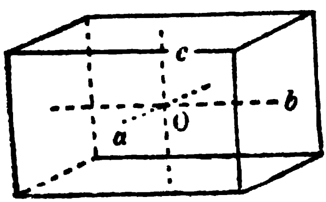



Fig. VI.6

(2) **The monoclinic system.** It is referable to three unequal crystallographic axes, one of which is at right angles to the other two. It has got a single 2-fold axis of symmetry or a single plane of symmetry (Fig. VI-5).

* In Figs. VI-4 to VI-10 the crystallographic axes are shown by the dotted lines ($a$,$b$,$c$,) passing through the origin O.

(3) **The orthorhombic system.** It is referable to three unequal crystallographic axes at right angles to each other. It has got two planes of symmetry at right angles or

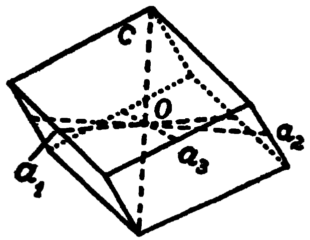



Fig. VI-7

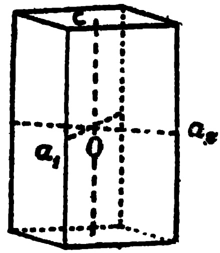



Fig. VI-8

three two-fold axes of symmetry at right angles to each other. Crystal aragonite ($CaCO_3$) belongs to this system (Fig. VI-6)

(4) **The trigonal** or **rhombohedral system.** It has got a single three-fold axis of symmetry and is referable to three equal crystallographic axes equally inclined to the three fold symmetry axis. Examples of this system are calcite ($CaCO_3$), quartz ($SiO_2$), and tourmaline ($H_6Na_2Fe_4B_6Al_3$ $Si_{12}$ $O_{63}$). We shall have to refer to these trigonal crystals frequently in this chapter.

(5) **The tetragonal system.** It has a single four-fold axis of symmetry. It is referable to three crystallographic axes mutually at right angles, two of them being equal (Fig. VI-8)

(6) **The hexagonal system.** It is referable to two equal crystallographic axes inclined at 120° and a third unequal crystallographic axis at right angles to the other two. It has a single six-fold axis of symmetry (Fig. VI-9).

(7) **The cubic system.** It is referable to three equal crystallographic axes at right angles to each other. It has got four three-fold axes of symmetry in the direction of the

diagonals of a cube. Optically this crystal is isotropic (Fig. VI-10).

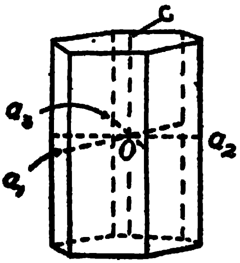



Fig. VI-9

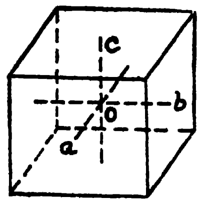



Fig. VI-10

**The calcite crystal.** The Iceland spar which is the transparent variety of calcite crystallises in many forms which by cleavage resolve into rhombohedral form. It has a single three fold or trigonal axis such that, if the crystal is rotated through an angle of $2\pi/3$ or 120° about this axis, then it is brought into self-coincidence. The rhombohedron is bounded by six parallelogram faces the angles of which are 101° 55′ and 78° 5.′ Two diametrically opposite corners A or B (Fig. VI-12) of the crystal are bounded by three obtuse plane angles each equal to 101° 55′.A line drawn through one of these corners and equally inclined to the three bounding faces is the axis of trigonal symmetry.

**The optic axis.** Any line drawn through a calcite crystal parallel to this trigonal axis possesses important optical properties to be described below. Any line drawn parallel to the trigonal axis is called the *optic axis* of the calcite crystal. Thus optic axis is not a fixed line but only a direction parallel to fixed line which is the axis of trigonal symmetry in the ease of calcite crystal. Theoretically, tetragonal and hexagonal crystals should possess similar optical properties like calcite, but transparent varieties of such crystals are very rare.

**Uniaxial and biaxial crystals.** Crystals of trigonal, tetragonal, and hexagonal symmetry possess only one optic axis. Hence they are called uniaxial crystals. Crystals of the orthorhombic,

monoclinic and triclinic systems have two directions each of which corresponds to the optic axis of uniaxial crystals. Such crystals are, therefore, called biaxial crystals. In this chapter we shall study the optical properties of uniaxial crystals only.

**The quartz crystal.** Ideal shape of the quartz crystal is is shown in Fig. VI-1. It belongs to the trigonal system. Its trigonal axis is the line joining the points A and B of the crystal in Fig. VI-1. The optic axis of the crystal is a direction through the crystal and parallel to this trigonal axis. Quartz crystal is transparent to visible and ultra-violet light.

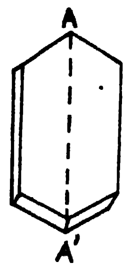


Fig. VI-11

**The tourmaline crystal.** By cleavage a crystal of tourmaline can be reduced to the shape shown in Fig. VI-11. It is slightly greenish in colour. The direction of the optics axis is any line through the crystal parallel to the line joining the points A and A′ in Fig. VI-11.

**A principal section** of a crystal in optical mineralogy, is a plane parallel to a specific plane containing the optic axis of the crystal. In the case of a calcite crystal, with faces bounded by equal sides the principal section is a plane passing through the shorter diagonal BD of the face ABCD (Fig. VI-12). In the calcite crystal ABCDEFGH shown in Fig. VI-12 any plane parallel to the plane BDEG is a principal section of the crystal, the line DK being the optic axis of the crystal through the point D. Evidently, the principal section cuts the calcite crystal in a parallelogram, the angles of the parallelogram being 71° and 109°. Any plane parallel to the plane ACHF which is perpendicular to the principal section will be referred to in this book as *normal section* of the crystal.

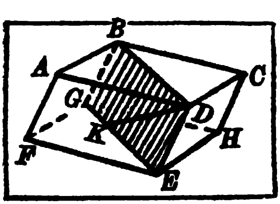


Fig. VI-12

**43. Double Refraction or Birefringence.** The first observation of the phenomenon of double refraction of which we have any record was made by Erasmus Bartolinus in the year 1669. He found that if a black dot A (Fig. 13*a*) marked on a white paper is covered with Iceland spar and viewed normally through the upper surface of the crystal, then instead of one dot two dots E and O are generally observed. The characteristics of these images are summarised below.

In Figs. VI-13 *a,b,c*, the broken lines show the positions of the optic axis in different orientations of the crystal.

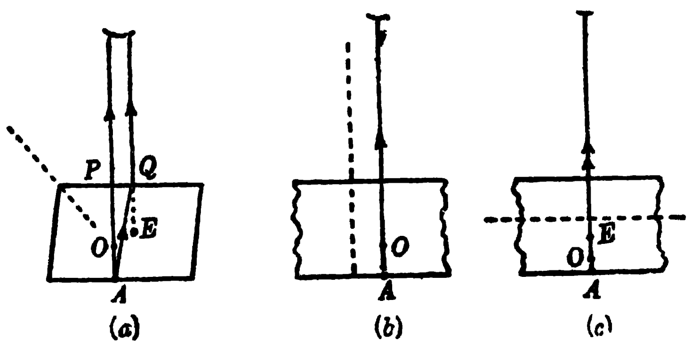



Fig. VI-13

(1) *One of these images O lies on the normal to the surface passing through A and the other E is slightly displaced both laterally and vertically with respect to O.*

Evidently, a ray entering the crystal breaks up into two rays which emerge directions parallel to one another. Thus fact was established by Huygens by the following experiment :

**Experiment.** Cover one face of the crystal by a sheet of black paper provided with small aperture (Fig. 14) and allow a parallel beam of light to be incident normally on this face of the crystal. The emergent beam is found to consist of two parallel pencils displaced laterally with respect to one another.

One of these emergent pencils AO is only a continuation of the incident pencil.

**The ordinary and the extra-ordinary rays.** This phenomenon of breaking up of a ray into two rays inside a crystal is called *double refraction* or *birefringence*. The ray AO is called the *ordinary ray* and the ray AE is called the *extra-ordinary ray*.

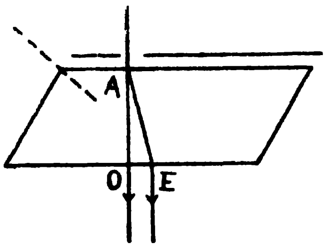


Fig. VI-14

(2) *The lateral separation between the emergent rays is directly proportional to the thickness of the crystal traversed by the rays.*

In Fig. 13(*a*) the apparent depth PO of the image O is less than the apparent depth QE of the image E.

Since, *Refractive index*$=\dfrac{\textit{Real depth}}{\textit{Apparent depth}}$

refractive index of the extra-ordinary ray EQ is

$$n_e=\frac{AP}{QE}$$

and that of the ordinary ray AP is

$$n_o=\frac{AP}{OP}$$

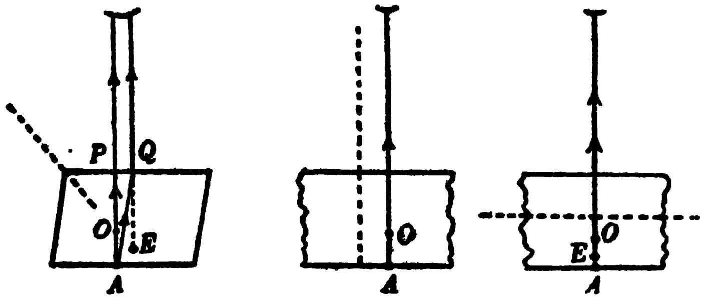
Hence $$\frac{n_e}{n_o}=\frac{OP}{QE}=\frac{v_o}{v_e}$$

where $v_o$ and $v_e$ are the velocities of the ordinary and the extra-ordinary rays respectively inside the crystal. Since OP$<$QE, $v_o<v_e$. Hence

(3) *in a calcite crystal, the velocity of the ordinary ray is less than the velocity of the extra-ordinary ray.*

If a plate of quartz crystal is placed on a black dot as in Fig. 15(a), the apparent depth QE of the extra-ordinary image is found to be less than that PO of the ordinary image. Hence

(4) *in a quartz crystal the velocity of the ordinary ray is greater than the velocity of the extra-ordinary ray.*




(a) (b) (c)

Fig. VI-15

**Positive and negative crystals.** Crystals like calcite in which the extra-ordinary ray moves with higher velocity are called negative crystals. Crystals like quartz in which ordinary ray moves with higher velocity than the extra-ordinary ray are called positive crystals. Double refraction in a positive crystal is called **positive birefringence** and that in a negative crystal is called **negative birefringence.**

If a parallel plate of calcite crystal is so cut that its optic axis is perpendicular to the two parallel refracting faces and if such a plate is placed on the black dot, then there would be only one image which is the ordinary image of the dot and there is only one emergent ray which is the ordinary ray (Fig. VI-13*b*). Hence

(5) *a ray incident on the crystal parallel to the optic axis is not doubly refracted. In other words, the optic axis is the direction of single wave velocity.*

The same conclusion also holds for positive crystals (Fig. VI-15*b*). In some books, the optic axis of a crystal is also called the *principal crystallographic axis.*

If the refracting faces of a parallel slab of calcite crystal is parallel to the optic axis, a normal line of vision would apparently yield only one image. But by using a suitable focussing device such as a short focus microscope, two images one above

the other will be seen. (See Fig. 13*c* for negative crystal and Fig. 15*c* for positive crystal). Hence

(6) *in a direction perpendicular to the optic axis, the ordinary and the extra-ordinary rays move in the same direction with different velocities.*

If the crystals in Figs 13*a* and 15*a* be rotated about the ordinary ray as axis, then the ordinary image remains fixed, while the extra-ordinary image rotates about the ordinary image *at the same rate*, their lateral separation remaining constant. Evidently,

(7) *both the ordinary and the extra-ordinary rays lie in a plane fixed relatively to the crystal.*

In the cases discussed in Figs. 13 and 15, the planes of diagrams are the principal sections of the crystal. Both the ordinary and extra-ordinary rays as well as the incident ray lie in this plane. As will be seen presently, *the ordinary ray, the extra-ordinary ray and the incident ray lie in the plane of incidence only when the plane of incidence coincides with the principal section of the crystal.* In the particular cases shown in Figs. 13 and 15 both the ordinary and the extra-ordinary images lie on a line parallel to the shorter diagonal of the crystal face having equal sides (Fig. VI-16) In general, the plane of refraction for the extra-ordinary ray is not the same as the plane of incidence: neither for the extra-ordinary ray the sine of the angle of incidence bears a constant ratio to the sine of the angle of refraction. In other words, the extra-ordinary ray does not, in general, obey any of the laws of refraction ; hence the name extra-ordinary. The ordinary ray, however, obeys both the laws of refraction under all conditions.

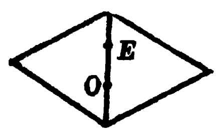


Fig. VI-16

**Principal plane of the ordinary ray** is the plane containing the optic axis and the ordinary ray.

**Principal plane of the extra-ordindry ray** is the plane containing the optics axis and the extra-ordinary ray.

**44. Huygens' Theory of Double Refraction.** In order to explain the phenomena of double refraction in uniaxial crystals, Huygens used his principle of secondary wavelets with slight modification. According to him

*Each point inside a crystal disturbed by light waves is the source of two secondary wavelets one of which is a sphere and the other is an ellipsoid of revolution round the optic axis. The sphere and the ellipsoid have a common centre which is the point of disturbance and they touch one another on the optic axis.*

In the case of a negative uniaxial crystal the sphere lies inside the ellipsoid (Fig. VI-17) and in the case of a positive uniaxial crystal the ellipsoid lies inside the sphere (Fig. VI-18).

The cross sections of the wave surfaces by three mutually

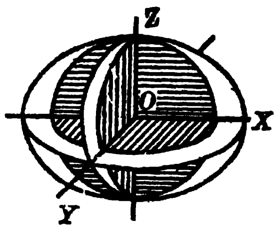


Fig. VI-17

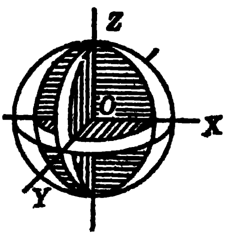


Fig. VI-18

perpendicular planes XOZ, ZOY, YOX are shown in Figs. VI-17 and VI-18, the line OZ being the optic axis.

**Construction for refracted wave-fronts.** For the construction of the ordinary and the extra-ordinary refracted wave-fronts, the same procedure as that used for refraction from one isotropic medium to another may be adopted (See General Introduction, P 15).

Let a plane wave-front AC (Fig. VI-19) initially moving in air be incident at the point A on the crystal surface PB at angle $i$. In the particular case shown in Fig. VI-19, the plane of incidence contains the optic axis DF. At the instant under consideration the point A of the crystal is just on the point of being disturbed. By the time the point C on the wave-front AC reaches the point

B of the crystal, the ordinary wave-front already generated at A expands to a sphere of radius AO given by

$$\frac{CB}{AO}=\frac{v_a}{v_o}=n_o$$

where $v_a$ and $v_o$ are the velocities of disturbance in air and in medium of the crystal respectively and $n_o$ is the refractive index of the ordinary ray in the crystal.

Let the optic axis of the crystal cut the ordinary spherical wave-front at the points F and D. Then an ellipse described with FD as the minor axis would represent the position of the wave-front at the instant when the point B

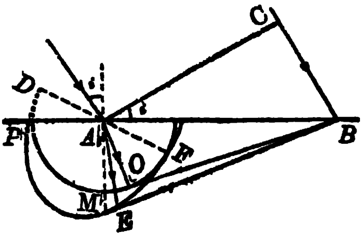



Fig. VI-19

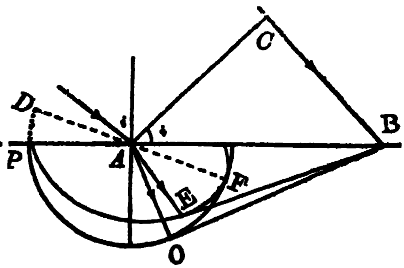
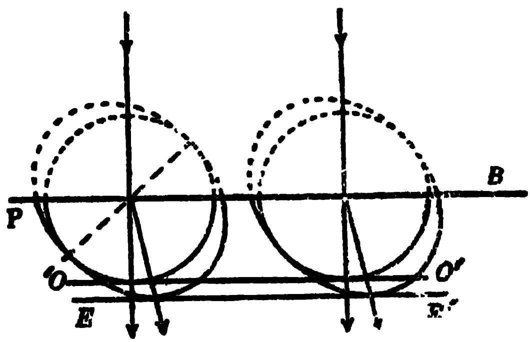



Fig. VI-20

is just on the point of being disturbed. The tangents BO and BE would then give the positions of the ordinary and the extra-ordinary wave-fronts. The lines AO and AE then represent the ordinary and the extra-ordinary rays respectively.




iFg. VI-21

The method of construction for the ordinary and the extra-ordinary wave-fronts in the case of positive crystals is shown in Fig. VI-20. It will be noticed that in the case of negative crystals, the extra-ordinary ray deviates more towards the normal than the ordinary ray. With positive crystals, the reverse is the case.

**Propagation of plane wave-fronts in a uniaxial crystal.** The method of construction for refracted wave-fronts in a uniaxial crystal explains the results of observation represented in a different way by Figs. 13 to 15. Let us consider the case of a parallel bundle of rays incident normally on a crystal surface. Let the plane of incidence contain the optic axis which is inclined to the direction of the incident ray shown by the dotted straight line in Fig. VI-21. The figure shows that even at normal incidence the incident ray breaks up into ordinary and extra-ordinary rays, and there are two refracted wave-fronts the ordinary OO′ and the extra-ordinary EE′.

In Fig. VI-22, both the incident ray and the optic axis are perpendicular to the refracting face of the crystal. In this case the tangent plane or the envelope of the secondary wave-

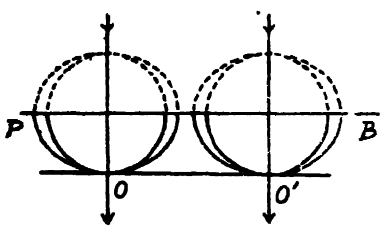



Fig. VI-22

fronts touch both the sphere and the ellipse at their common points. Hence there is only one refracted wave-front and this is the ordinary wave-front.

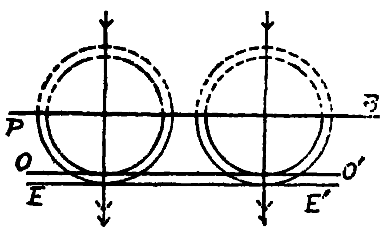



Fig. VI-23

In Fig. VI-23, the optic axis is parallel to the refracting surface of the crystal and is perpendicular to the plane of the paper. Hence the sections of the wave surfaces by the plane of incidence

are two concentric circles, and there are two refracted wave-fronts moving with two different velocities in the same direction.

In Fig. VI-24, the optic axis is parallel to the refracting surface of the crystal and also to the plane of the paper. Hence the section of the wave-fronts by the plane of incidence would consist of a circle and an ellipse. In this case, too, there are two refracted wave-fronts moving in the same direction with different velocities.

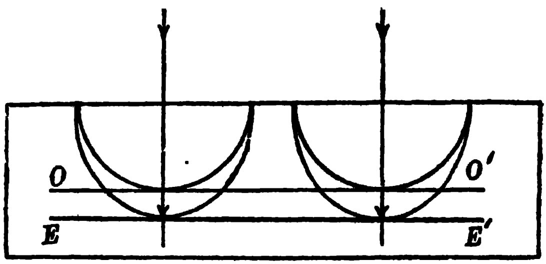



Fig. VI-24

In all the cases considered above the optic axis lies in or perpendicular to the plane of incidence and both the ordinary and extra-ordinary rays lie in the plane of incidence. If the optic axis does not lie in the plane of incidence, the extra-ordinary ray would not lie in the plane of incidence. This case cannot, of course, be represented by a simple sectional diagram.

**45. Refractive Index of Crystals.** It would appear from the method of construction that the ordinary refracted ray AO (Fig. VI-19) is the line joining the point of incidence A to the point of contact O of the ordinary wave envelope BO with the ordinary wave surface. Similarly, the extra-ordinary refracted ray AE is the line joining the point A to the point of contact E of the extra-ordinary wave envelope BE with the extra-ordinary wave front. Obviously the extra-ordinary ray would not lie in the plane of incidence unless the plane of incidence contains the optic axis as in Fig. VI-25. Since the ordinary wave-front is a sphere, the plane of refraction for the ordinary ray would always be the same as the plane of

incidence. Let us now try to deduce expressions for the ordinary and extra-ordinary refractive indices.

**Refractive index of ordinary ray.** From geometry of the figure VI-25 the refractive index of the ordinary ray is

$$n_0 = \frac{\sin i}{\sin r_0} = \frac{\sin \angle CAB}{\sin \angle ABO} = \frac{CB/AB}{AO/AB} = \frac{CB}{AO} \quad \ldots \quad (41{\cdot}1)$$

or

$$n_0 = \frac{v}{v_0} \quad \ldots \quad (41{\cdot}2)$$

where $v_0$ is the velocity of the ordinary refracted ray in the crystal and $v$ is the velocity in vacuo. $n_0$ is constant for all possible angles of incidence. Further, the refracted ordinary ray always lies in the plane of incidence. Hence the ordinary ray obeys both the laws of refraction.

**Refractive index of the extra-ordinary ray.** The above method of defining the refractive index is not generally applicable to the case of extra-ordinary ray, since the extra-ordinary ray AE and the wave normal AN to the extra-ordinary wave envelope BE

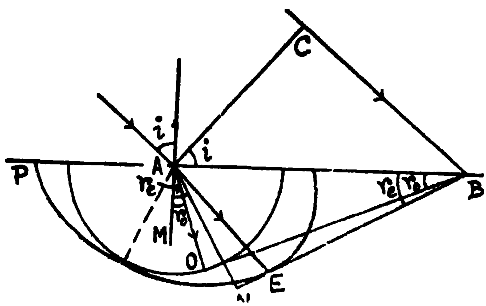



Fig. VI-25

are not coincident except in certain particular cases. This means that the direction of propagation of the extra-ordinary ray (that is, the direction AE) is not the same as the direction of propagation of the extra-ordinary wave-front which is the direction of the normal AN to the extra-ordinary wave envelope BE.

**Wave velocity and ray velocity.** The rate of increase of the line AE with respect to time is called the ray velocity ;

while the rate of increase of the normal AN to the wave-front with respect to time is called the wave velocity.

Hence to define the extra-ordinary refractive index, the angle of refraction is taken to be the angle between the normal AM at the point of incidence and the wave normal AN to the extra-ordinary wavefront and not the angle between the normal AM and the extra-ordinary ray AE. Accordingly, if the *angle of refraction* NAM *of the wave normal* (as we shall call it) be represented by $r_e$, then, the refractive index of the extra-ordinary ray is

$$n_e = \frac{\sin i}{\sin r_e} = \frac{\sin \angle CAB}{\sin \angle ABN} = \frac{CB/AB}{AN/AB} = \frac{CB}{AN} \quad \dots \quad (41\cdot3)$$

Hence
$$n_e = \frac{v}{v_e} \quad \dots \quad (41\cdot4)$$

where $v_e$ is the velocity of the extra-ordinary wave front inside the crystal. In other words,

$$n_e = \frac{\textit{velocity of wave-front in vacuo}}{\textit{velocity of extra-ordinary wave-front in crystal}} \quad \dots (41\cdot5)$$

Hence *the refractive index of the extra-ordinary ray in any direction is inversely proportional to the length of the wave normal* AN (from Eqn. 41·3)

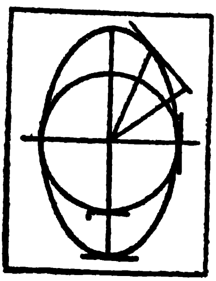


Fig. VI-26

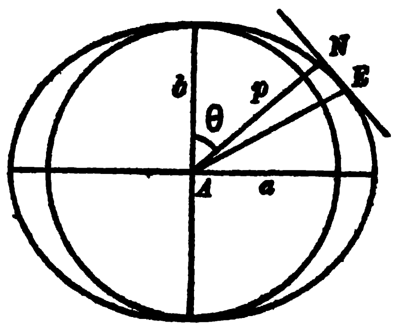



Fig. VI-27

It is obvious that the extra-ordinary refractive index depends on the angle of incidence and also on the orientation of the optic axis relative to the plane of incidence. Accordingly the extra-ordinary refractive index lies between a maximum and a minimum value, the refractive index being minimum when

the extra-ordinary wave-front in the crystal is perpendicular to the major axis and it is maximum when it is perpendicular to the minor axis (by Eqn. 41·3) Since in the case of negative crystals, the sphere lies inside the ellipsoid, it follows from Figs. VI-25 and VI-26 that the maximum refractive index of the negative crystals is equal to the ordinary refractive index.

Let these maximum and minimum refractive indices be represented by $n_0$ (the refractive index of the ordinary ray) and $n_E$ respectively. If $a$ and $b$ are the semi-major and semi-minor axes of the ellipsoid, then

$$a \propto \frac{1}{n}, \text{ and } b \propto \frac{1}{n_0}$$

It is proved in Books on Conic Sections that

$$p^2 = a^2 \sin^2\theta + b^2 \cos^2\theta \qquad \dots \quad (41\cdot6)$$

where $p$ is the length of the normal from A (Fig. VI-25) on the tangent plane to the ellipse and $\theta$ is the angle between the normal and the minor axis, that is, the angle between the direction of wave-propagation and the optic axis (in the case of uniaxial crystal). If $n_e$ be the refractive index in this direction AN making an angle $\theta$ with the optic axis, then

$$\frac{1}{n_e^2} = \frac{\sin^2\theta}{n_E^2} + \frac{\cos^2\theta}{n_0^2} \qquad \dots \quad (41\cdot7)$$

From this equation knowing $n_0$ and $n_E$, the extra-ordinary refractive index $n_e$ in any direction can be determined. *The minimum value of refractive index $n_E$ of the extra-ordinary ray in the direction of the major axis of the ellipse, that is, in the direction perpendicular to the optic axis is called extra-ordinary refractive index of the negative crystal.*

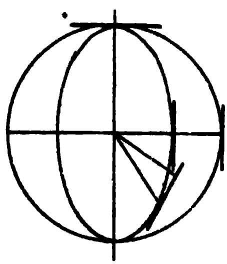


Fig. VI-28

Hence the extra-ordinary wave-front in a negative crystal is an ellipsoid of revolution round the optic axis with its major and minor axes inversely proportional to minimum and maximum refractive indices restively.

**In the case of positive crystals,** the ellipsoid lies inside the sphere (Fig. VI-28). Hence the extra-ordinary refractive index

has its minimum value in the direction of major axis of the ellipse. This is also equal to the ordinary refractive index of positive crystal. The maximum value of refractive index for the extra-ordinary ray corresponds to the direction of minor axis of the ellipse, that is, it lies in the direction perpendicular to the optic axis. Hence, in the case of positive crystals

$$a \propto \frac{1}{n_O}, \quad \text{and} \quad b \propto \frac{1}{n_E}$$

where $n_O$ and $n_E$ are the minimum and maximum refractive indices respectively. If $n_e$ is the refractive index in any direction AN $(=p)$ making an angle $\theta$ with the minor axis, we have

$$n_e \propto \frac{1}{p} \quad \text{(from Eqr. 41·3)}$$

Hence substituting the values of $p$, $a$, and $b$ in Eqn. 41·6, we get

$$\frac{1}{n_e^2}=\frac{\sin^2\theta}{n_O^2}+\frac{\cos^2\theta}{n_I^2} \qquad \dots \quad (41{\cdot}8)$$

*This maximum value of refractive index in the direction perpendicular to the optic axis of a positive crystal is called its extra-ordinary refractive index.*

Knowing the values of $n_O$ and $n_E$, the refractive index in any direction making an angle $\theta$ with the optic axis can be calculated with the help of Eqn. 41.8.

**Refractive indices of uniaxial crystals.** The ordinary and the extra-ordinary refractive indices of a uniaxial crystal can be determined with the help of a prism spectrometer by the usual method. For this purpose a prism is cut from

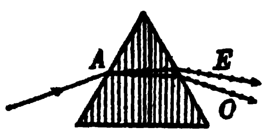


Fig. VI-29

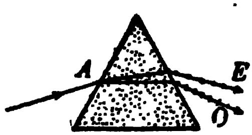


Fig. VI-30

crystal in either of the two ways shown in Figs. VI-29 and VI-30. In Fig. VI-29. the optic axis is perpendicular to refracting edge and bisects the refracting angle of the prism. In Fig. VI-30 the optic axis is parallel to the refracting edge of the prism. A ray incident on the prism (Fig. VI-29) at the point A is broken up into an ordinary and an extra-ordinary rays. If the prism is placed in the position of minimum deviation with respect to the

extra-ordinary ray, then the path of this ray within the prism will be parallel to the major axis of the ellipse and the refractive index determined by the relation

$$n_E = \frac{\sin \frac{1}{2}(D_m + A)}{\sin \frac{1}{2}A}$$

would give the extra-ordinary refractive index. The ordinary refractive index can be determined by placing the prism in the position of minimum deviation for the ordinary ray.

In the case of the prism shown in Fig. VI-30, the plane of incidence cuts the wave surfaces in concentric circles. Hence the extra-ordinary ray obeys both the laws of refraction, so that, both the ordinary and extra-ordinary refractive indices can be determined easily in the usual way with the help of the prism spectrometer.

**Absorption power of crystals.** On transmission through a crystal, the ordinary and the extra-ordinary rays are absorbed to different extents by the crystal. Thus, a plate of tourmaline (1 to 2 mm. thick) absorbs the ordinary ray completely and there is only one emergent ray and that is the extra-ordinary ray. A plate of agate cut perpendicularly to its cleavage plane completely absorbs one of the two emergent rays, if the plate is of sufficient thickness.

**42. Polarisation of Light. Discovery of Malus.** The phenomenon of double refraction described above is always associated with lack of symmetry of the doubly refracted rays round the directions of propagation of these rays. This was first observed by Malus. An announcement of a proposed prize by the Paris Academy for a mathematical theory of double refraction directed Malus towards the investigation of the phenomenon of double refraction. While engaged in this problem at his house he chanced to look at the reflection of the setting sun in a window of the Luxembourg palace through a calcite crystal and was astonished to find that for certain orientation of the crystal only one ray instead of two was transmitted. In one complete rotation of the crystal, the ordinary and the extra-ordinary ray appeared and disappeared twice alternately. In the meantime the sun had set and Malus had to repeat his

experiment with light from a candle flame reflected at the surface of water and then from a glass surface and he examined this twice reflected light through the calcite crystal. He then changed the order of the experiment. He first allowed the light to be transmitted through calcite crystal and then allowed the transmitted light to be reflected at a glass plate getting the same result in every case.

It was found by Malus that light reflected from the surface of water at an angle of $52^\circ 45'$ was transmitted through the calcite crystal as ordinary ray when the principal section of calcite was parallel to the plane of incidence. When the principal section was perpendicular to the plane of incidence, only the extraordinary ray was transmitted.

Such modifications of light by reflection was called **polarisation** by Malus.

**Experiment on polarisation by reflection.** The most convenient apparatus used for this purpose is the Norrenberg polariscope shown in Fig. VI-31. It consists of a glass plate $M_1$ which can be rotated about a horizontal axis. At the top of the instrument there is a silvered mirror $M_2$ hinged to two vertical uprights fitted to a collar that can be rotated about a vertical axis, the mirror $M_2$ itself being capable of rotation about a horizontal axis passing through the two hinges.

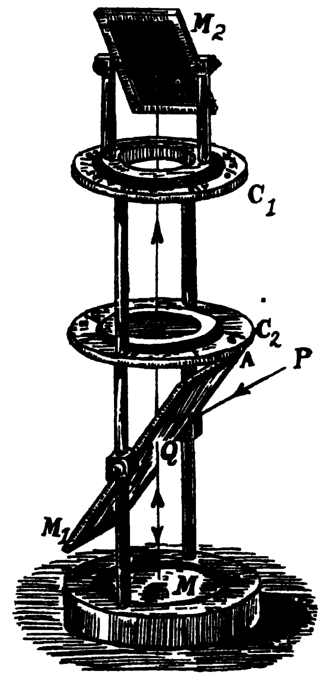


Fig. VI-31

The base of the instrument carries a fixed horizontal mirror M. The instrument is also provided with a removable glass stage $C_2$ placed between the two reflectors $M_1$ and $M_2$ for studying the optical behaviour of transparent bodies illuminated by polarised light.

A beam of light moving in the direction PQ is polarised by reflection at the mirror $M_1$; it is then reflected normally upwards by the mirror $M_2$ and falls on the mirror M. As the mirror $M_2$ is rotated about the vertical axis, the intensity of light reflected from $M_2$ passes through alternate maximum and minimum values. The intensity is maximum when the angle between the two planes of incidence at the two mirrors $M_1$ and $M_2$ is 0° or 180°, and it is minimum when the angle between the two planes of incidence is 90°, that is, when mirrors are *crossed*, as it is technically called. The two mirrors $M_1$ and $M_2$ are called the **polariser** and the **analyser** respectively.

**Brewster's law.** It is found that the minimum intensity of the rays reflected from $M_2$ when the mirrors $M_1$ and $M_2$ are crossed is nearly equal to zero (or, the polarisation is maximum) for a particular angle of incidence which is called the **polarising angle.** This polarising angle depends on the refractive index of the reflecting medium given by

$$n = \tan p$$

where $p$ is the polarising angle. This law is known as Brewster's law. This polarising angle thus depends on the refractive index and it is, therefore, different for different colours.

An immediate consequence of Brewster's law is that under the condition of maximum polarisation by reflection the reflected and refracted rays must be perpendicular to one another. For, if $r$ be the angle of refraction corresponding to the polarsing angle $p$, then

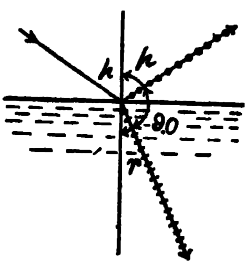



Fig. VI-32

$$n = \frac{\sin p}{\sin r} = \tan p = \frac{\sin p}{\cos p}$$

Hence $\sin r = \cos p$

or $r + p = 90^\circ$

This result obtains for glass when the angle of incidence is nearly 57°. In other words the polarising angle for glass is 57°.

At the polarising angle, that is, when the reflected and refracted rays are at right angles to one another, the reflected

beam is *almost* completely polarised, complete polarisation being probably prevented by contamination of the glass surface.

**The cosine square law of Malus.** This law states that *the intensity of the beam reflected twice from two reflecting surfaces is proportional to the square of cosine of the angle between the planes of incidence at the two reflecting surfaces.*

This law may be regarded as the climax of Malus' discoveries, for, it immediately leads to the conception of mechanical nature of the asymmetry of light rays which has been called polarisation. We now proceed to prove that the cosine-square law of Malus is compatible with the asymmetry in the direction of vibration of ether particles round the ray which constitutes light. Naturally, light vibrations cannot be longitudinal, for, with longitudinal waves the question of asymmetry in the direction of vibration does not arise Hence light vibrations must be transverse. Hitherto we had no occasion to refer to the direction of vibration in light waves, since the phenomena of interference and diffraction have nothing to do with the direction of vibration ; they depend only on the superposition of displacements.

**Natural light and polarised light.** As a starting hypothesis we have to assume that in natural light the direction of vibration of ether particles is transverse to the direction of propagation of

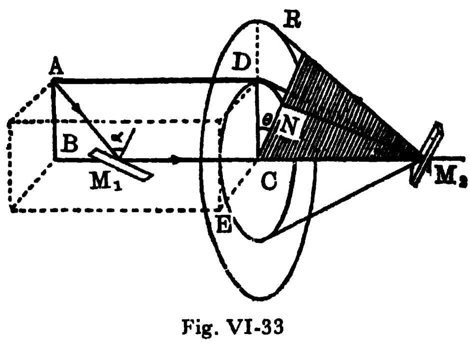



Fig. VI-33

waves, but this direction of vibration at a point changes many times per second. When a beam of light is polarised by reflection or by refraction through a crystal, the directions of trans-

verse vibration are all confined to one particular plane passing through the direction of propagation and the light is said to be **plane-polarised.**

**Representation of natural and polarised light.** For the sake of diagrammatic representation we shall represent light vibrations in a plane by short lines at right angles to the ray in the plane and light vibration perpendicular to a plane by dots on the ray.

In Fig. VI-33. let $M_1$ be the polarising mirror and $M_2$ the analysing mirror of the Norrenberg polariscope placed horizontally and let the plane $ADM_2CB$ be the plane of incidence at the mirror $M_1$. The analysing mirror $M_2$ can be rotated about the incident ray $BCM_2$ as axis. Let $M_2N$ be the position of the normal to the mirror $M_2$ at an instant, so that, the plane $CNM_2$ is the plane of incidence at mirror $M_2$.

Let $\theta$ be the angle between the two planes of incidence at the instant under consideration. As $M_2$ rotates about the incident ray as axis, the normal M N describes a cone of semi-vertical angle $\alpha$ which is equal to the angle of incidence at the mirrors $M_1$ and $M_2$. The reflected ray $M_2R$ necessarily describes another cone of semi-vertical angle equal to $2\alpha$. Let us assume that *the vibration which can be reflected is that which is executed only in a direction perpendicular to the plane of incidence.* Then the vibration in the polarised light after reflection at the mirror $M_1$ is executed in a direction parallel to CE. If $a$ is the amplitude of vibration in the beam incident on the mirror $M_2$ then the vibration in the beam reflected from $M_2$ must be the component of the incident amplitude in a direction perpendicular to the plane of incidence $CNM_2$ and this is equal to $a\cos\theta$. Since intensity is proportional to the square of amplitude, the intensity of the polarised beam reflected from $M_2$ is $a^2\cos^2\theta$ which agrees with the cosine square law of Malus.

**Polarisation of the refracted beam.** When a beam of light is reflected from the surface of a transparent medium like glass the rest of the light that is not reflected must be refracted. If the incident beam is unpolarised, it must contain two equal proportions of light with their directions of vibration parallel

and perpendicular to the plane of incidence. Since the reflected light contains some of the perpendicular vibrations only, the refracted portion must contain the parallel components together with the rest of the natural light. Thus both the reflected and refracted components are polarised; the refracted compenent being a mixture of polarised and natural light, it is only partially polarised. The direction of vibration in the reflected beam is perpendicular to the plane of incidence, while that in the partially polarised refracted beam is parallel to the plane of incidence.

**Pile of plates.** By reflecting a beam of light successively from a pile of parallel plates of glass, the percentage of polarisation in the transmitted beam can be much increased, because at each reflection the transmitted beam would lose more and more of the perpendicular components. By increasing the number of plates and reflecting the light at the polarising angle, the refracted beam can also be completely polarised.

**Polarisation by double refraction.** We have seen that a ray of light incident normally on the surface of calcite

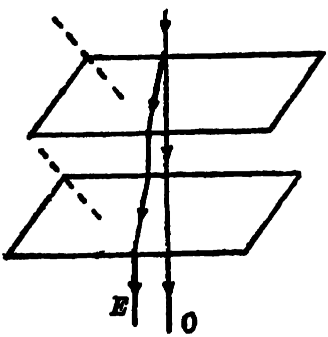



Fig. VI-34

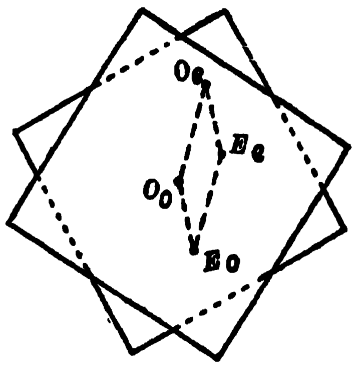



Fig. VI-35

crystal is broken up into two components, ordinary (O) and extra-ordinary (E) both the ordinary and the extra-ordinary rays lying in the principal section of the crystal. If a second similar crystal of the same substance is placed in the path of the

emergent rays, with their principal sections placed parallel to one another, there would be two emergent rays as before, only their lateral separation would increase (Fig. VI-34).

On rotating the second crystal with respect to the first, each of the ordinary and the extra-ordinary components again breaks up into two components, the ordinary component O giving rise to one ordinary ray ($O_o$) and one extra-ordinary ray ($O_e$). The direction of the component ($O_o$) is only prolongation of the ordinary component from the first crystal and the component $O_e$ is displaced in the direction of shorter diagonal of the second crystal. Similarly, the E-component on entering the second crystal is broken up into two components, ordinary ($E_o$) and the extra-ordinary ($E_o$). The component $E_o$ is the prolongation of the extra-ordinary component from the first crystal while the component ($E_e$) is displaced in the direction of shorter diagonal of the second crystal. Hence the components $O_o$ and $E_e$ lie on the shorter diagonal of the first crystal and the components $O_e$ and $E_e$ lie on the shorter diagonal of the second crystal; in other words, the four emergent rays intersect a screen at points which form the four corners of a parallelogram of which the two adjacent sides are parallel to the two shorter diagonals of the two crystals (Fig. VI-35).

The intensities of these four images depend on the orientations of the two crystals relative to one another.

For the sake of simplicity we shall suppose in the following paragraph that one of the rays (the ordinary, say) coming out of the first crystal is stopped out by a diaphragm. Let us suppose that the second crystal is kept fixed while the first crystal with the diaphragm is rotated round the incident ray.

**Intensity considerations.** Experiment shows that when the principal sections of the two crystals are parallel there is only one ray of maximum intensity and this is the extra-ordinary ray. As the first crystal is rotated the single ray breaks up into two, one of which is ordinary ($E_o$) and the other extra-ordinary ($E_e$) the extra-ordinary being more intense than the ordinary. When the angle between the principal sections is 45°, the two ra

are of equal intensity. Beyond 45°, the ordinary ray becomes more intense than the extra-ordinary ray. When the angle between the principal sections is 90°, the intensity of the extra-ordinary ray becomes zero and that of the ordinary ray becomes maximum. Beyond 90°, the intensity of the ordinary ray diminishes and that of the extra-ordinary ray increases till when the angle between the principal sections is 180°, there is only one ray, the extra-ordinary ray.

If the second crystal remains fixed and the first is rotated, the ordinary and the extra-ordinary ray remain fixed but they appear and disappear in turn twice in each revolution. If the first crystal remains fixed while the second rotates, then the extra-ordinary ray rotates round the ordinary ray and at the same time undergoes changes in intensity.

**Compatibility with idea of polarisation.** The results of experiments described above can be explained if we assume that both the ordinary and the extra-ordinary rays are polarised.

It follows from Malus' experiment (see P. 193) that light reflected from water surface at the polarising angle is transmitted as ordinary ray through calcite when the principal section of calcite is parallel to the plane of incidence. We have further assumed that the vibration in the reflected beam is perpendicular to the plane of incidence. It follows that *vibration in the ordinary beam must be in a direction perpendicular to the principal section. Similarly, the direction of vibration in the extra-ordinary ray must be parallel to the principal section.*

In the case under consideration the principal planes of the ordinary and the extra-ordinary rays are coincident with the principal section of the crystal, since the incident ray is normal to the surface of the crystal. In Fig. VI-37, QP is the direction of the extra-ordinary ray coming from the first crystal and incident at the point P of the second crystal. Let the principal section $PRSE_0$ of the first crystal make an angle $\theta$ with the principal section TXLM of the second crystal shown by the shadow diagram in Fig. VI-36. Let $E_0PYN$ be the normal section of

this cyrstal. Since the extra-ordinary ray vibrates in the principal section, the vibration of the incident ray QP is executed in the direction PR. Assuming that a uniaxial crystal can transmit only those vibrations which are executed in the principal and normal sections the incident vibration PR of amplitude $a$ must break up into two components— $a \cos \theta$ in the direction PX and $a \sin \theta$ in the direction PY. The component PX ($=a \cos \theta$) which lies in the principal section corresponds to the extra-ordinary ray $PE_e$ and the component PY ($=a \sin \theta$) which is executed in the normal section corresponds to the ordinary ray $PE_0$ (Fig. 38).

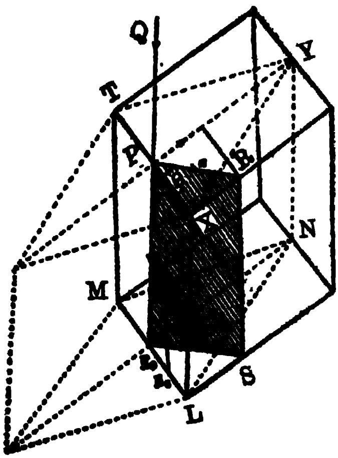



Fig. VI-36

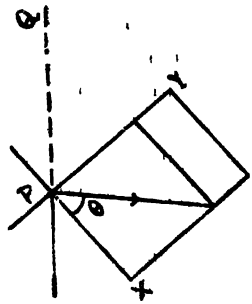



Fig. VI-37

Hence the intensity of the ordinary ray is $a^2 \sin^2\theta$ and that of the extra-ordinary ray is $a^2 \cos^2\theta$. As the first crystal is rotated the angle $\theta$ increases and, therefore, the intensity of the E-component diminishes and that of the O-component increases. When the angle $\theta$ is equal to 90° the intensity of the E-component is zero and that of the O-component is maximum. When $\theta$ is equal to 180°, the intensity of the O-component is zero and that of the E-component is maximum.

It follows that both the ordinary and the extra-ordinary rays obtained by double refraction are plane-polarised. Their planes

of vibration are perpendicular to one another, the ordinary ray vibrating in the normal section and the extra-ordinary ray vibrating in the principal section.*

**43. Methods of producing polarised Light.** As we have seen already there are two methods of production of plane-polarised light : (1) by the method of reflection, (2) by refraction through crystals. By reflection complete polarisation is very difficult to secure. Hence for laboratory and technical purposes polarisation by the method of reflection is seldom used. The simplest method of obtaining polarised beam is the method of double refraction. But with crystals two polarised beams are generally obtained and unless the crystals are sufficiently thick, the ordinary and the extra-ordinary beams of light would overlap. With a crystal of tourmaline this difficulty is partly removed, because the ordinary ray is completely absorbed by the crystal. But the polarised beam is coloured and due to its strong absorption power a tourmaline crystal cannot be used except with a strong source of illumination. The calcite crystals have special advantage in this respect in the sense that it is very transparent to visible light. Hence, if one of the doubly refracted beams could be removed, it would be the best device for obtaining plane polarised light.

**The Nicol's prism.** The Nicol's prism is made of calcite crystal in which the ordinary beam is removed by total reflection. A clear and flawless cleavage rhombohedron of calcite of length equal to three times the breadth is chosen for the purpose. Fig. VI-38 shows the principal section of the crystal. The natural faces of the crystal shown by dotted lines are inclined to each other at an angle of 70° 53′. By grinding and polishing, this inclination is reduced to 68°. The prism thus prepared is cut diagonally and perpendicularly to the principal section. After polishing the two cut surfaces, they are cemented together in their original positions with canada balsam.

*In some books a term *plane of polarisation* is very often used to indicate that the plane of vibration is perpendicular to the plane of polarisation. This only adds to the confusion of thought. Hence, this term has not been used in this book.

An incident ray moving in a direction parallel or nearly parallel to the length of the prism breaks up into an ordinary and an extra-ordinary ray, The refractive index of calcite for ordinary ray is 1·66 and for the extra-ordinary ray it is 1·48. Refractive index of transparent canada balsam cement is 1·55. Hence with respect to ordinary ray calcite is optically denser than canada balsam, but with respect to extra-ordinary ray canada balsam is optically denser than calcite. Hence the extra-ordinary ray is freely transmitted but the ordinary ray can be totally reflected if the angle of incidence at the balsam surface is greater than the critical angle. This critical angle for the ordinary ray is given by

$$\sin\theta = \frac{1\cdot55}{1\cdot65} = \cdot939$$

so that $\theta = 69^\circ 20'$

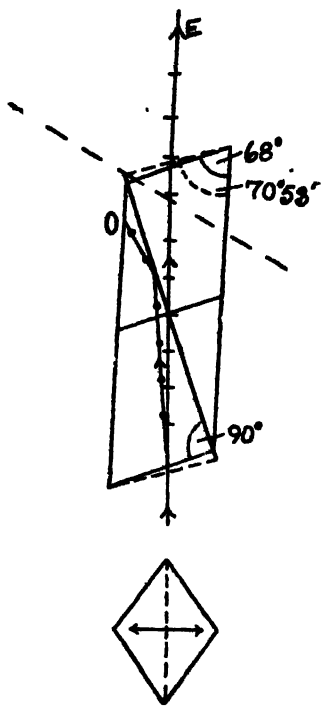


Fig. VI-38

It will appear from the geometry of the figure that the angle of incidence of the extra-ordinary ray moving parallel to the length of the prism at the point of incidence is 68°. Since the ordinary ray intersects the surface at a greater distance, the angle of incidence of the ordinary ray is greater than the critical angle. The ordinary ray is, therefore, totally reflected and passes out through the side of the prism and is absorbed by the non-reflecting mounting material which is usually cork painted dead black. Since the extra-ordinary ray vibrates in the principal section, the direction of vibration of the emergent ray from the prism is along the shorter diagonal of the end-face of the prism shown in the inset.

The inclined face of the prism results in loss of light by reflection and when used as analyser produces some displacement of the object in the field of view.

Many modifications of Nicol's prism have been introduced to increase the field of view, for making the field of view symmetrical, to decrease the amount of calcite required and also to reduce the slight elliptic polarisation caused by the end faces being inclined to the incident rays. The most commonly used Nicol's prism now-a-

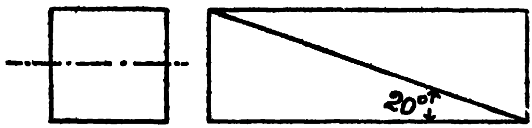



Fig. VI-39

days is the Glan-Thompson type (Fig. VI-39). In the original type of Nicol the field of view was 24°, while in Glan-Thompson type the field is increased to 40°. In Nicol's prism the cementing material canada balsam absorbs the ultra-violet rays. Hence such prism cannot be used for study of ultra-violet light.

**Foucault's prism.** It is evident that the less the critical angle, the smaller would be the length of the prism required to satisfy the condition of total reflection of the ordinary ray. Now the critical angle for ordinary and extra-ordinary rays moving from calcite into air are respectively 37°14′ and 42°23′, whereas the critical angle for ordinary ray moving from calcite into canada balsam is nearly 70°. Hence, if canada balsam is replaced by air, a much shorter length of calcite crystal would be sufficient to reflect the ordinary ray critically. This is the principle of Foucault's prism. But since refractive index of the extra-ordinary ray in calcite differs widely from that in air. most of the extra-ordinary rays is reflected at the air-surface and the intensity of the transmitted beam is very feeble.

**Rochon's prism.** It is sometimes necessary to have both the ordinary and the extra-ordinary rays separated by a wide angle. This is effected by a combination of two prism ABC and

ACD (Fig. VI-40) of calcite or quartz cemented with glycerin or castor oil. In one of these the optic axis is perpendicular to the face AB. In the other prism the optic axis is perpendicular to the plane ACD. They are cemented together so as to form a rectangular block.

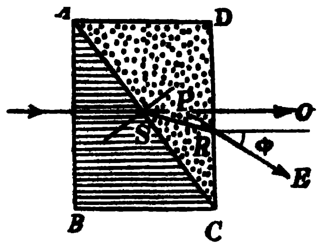



Fig. VI-40

A ray incident normally on the face AB of the prism ABC moves in the direction of the optic axis and accordingly it is not broken up into ordinary and extra-ordinary rays. But on entering the prism ACD, it breaks up into ordinary and extra-ordinary rays, the ordinary ray moving without deviation and the extra-ordinary ray suffering two deviations at the faces AC and CD.

To find the angle of divergence between the two emergent rays, let the refracting angle BAC of the prism be $A$, so that the angle of incidence on the interface AC is also $A$. Let the angle RSP be $\delta$, so that the angle of refraction in the prism ACD is $(A+\delta)$. If $v_0$ and $v_e$ are the velocities of the ordinary and the extra-ordinary rays in the prism then

$$\frac{\sin(A+\delta)}{\sin A}=\frac{v_e}{v_0} \qquad \ldots(43\cdot1)$$

Since $\delta$ is usually small, this becomes

$$\frac{\sin A+\delta\cos A}{\sin A}=\frac{v_e}{v_0}$$

or
$$1+\delta\cot A=\frac{v_e}{v_0}$$

or
$$\delta=\frac{v_e-v_0}{v_0}.\tan\ A \qquad \ldots(43\cdot2)$$

Because the two prisms are of the same material the velocity of the ordinary ray is the same in both.

Hence $$n_0 = \frac{\sin A}{\sin r_0} = \frac{v_0}{v_0} = 1$$

or $$A = r_0$$

Let $\phi$ be the angle of divergence between the emergent rays. Then, the angle of incidence on the face CD is $\delta$ and the angle of refraction is $\phi$, so that

$$\frac{\sin\phi}{\sin\delta} = \frac{C}{v_e} \qquad \ldots \quad (43.3)$$

where $C$ is the velocity in air. This gives for small value of $\delta$

$$\sin \phi = \delta . \frac{C}{v_e}$$

$$= \frac{C}{v_e}\left(\frac{v_e - v_0}{v_0}\right)\tan A$$

$$= \left(\frac{C}{v_0} - \frac{C}{v_e}\right)\tan A$$

or $$\sin \phi = (n_0 - n_e) \tan A \qquad .. \quad (43.4)$$

In the case of positive crystals like quartz the deviation of the emergent ray will be in the opposite direction.

**Wollaston's prism.** In this type of prism much wider divergence is obtained than in Rochon's prism. This also consists of a combination of two prisms (Fig. VI-41). In the first prism the optic axis is parallel to the refracting face and lies in the plane of the paper. But, in the other prism the optic axis is perpendicular to the plane of the paper. A ray incident normally on the face of the first prism moves inside the prism with two different velocities without separation (see Fig. VI-24). Let $v_0$ and $v_e$ be the velocities of ordinary and extra-ordinary rays inside the first prism. Inside a crystal there are only two possible directions of vibration

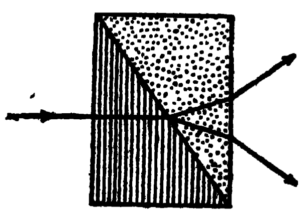


Fig. VI-41

(1) parallel to the optic axis (2) perpendicular to the optic axis. In traversing the first crystal the vibration giving rise to ordinary ray is executed in a direction perpendicular to its optic axis. This direction is perpendicular to the optic axis in the second crystal. This is because the optic axis in the two crystals are perpendicular to one another. Hence the ordinary ray in the first prism will travel as extra-ordinary ray in the second prism with velocity $v_0$ and similarly the extra-ordinary ray in the first prism will travel as ordinary ray in the second prism with velocity $v_e$.

As in the case of Rochon's prism the divergence of the ray is given by

$$\sin \phi_1 = (n_0 - n_e) \tan A. \quad \text{from Eqn. (43·4)}$$

Divergence of the other ray is obtained by interchanging $v_0$ and $v_e$ in Eqns. (43·2 and 43·3). Calculating in the same way as under Rochon's prism, the divergence $\phi_2$ of this ray is given by

$$\sin \phi_2 = (n_e - n_0) \tan A.$$

Hence $\phi_1$ and $\phi_2$ are of opposite signs and the divergence between the two emergent rays is double that in Rochon's prism.

**The Cornu polariscope.** A Wollaston's prism combined with a Nicol is known as Cornu polariscope. Since the two emergent rays from Wollaston's prism vibrate in directions which are perpendicular to one another, one or the other of the rays may be extinguished by rotating the Nicol in one direction or the other. In the two positions of extinction, the principal sections of the Nicol must be perpendicular to one another. Due to absorption by the Wollaston's prism the intensity of the two emergent rays are not equal. Since the Nicol only transmits rays vibrating in the principal section, the intensity of the two emergent beams may be made equal by rotating the Nicol through a proper angle.

Let $a$ and $b$ be the amplitudes of the ordinary and the extra-ordinary rays incident on the Nicol and let $\theta$ be the angle through which the Nicol is rotated from the position of extinction of the extra-ordinary rays to make the two intensities equal.

Then the component of the ordinary ray transmitted through the Nicol is $a \sin \theta$ and that of the extra-ordinary ray is $b\cos\theta$. Since the two transmitted components are equal,

$$a^2 \sin^2 \theta = b^2 \cos^2 \theta.$$

or

$$\frac{b^2}{a^2} = \tan^2 \theta. \quad \ldots \quad (43.5)$$

Since a small change in the angle of the Nicol produces a marked change in relative intensity, the Cornu polariscope may be conveniently employed for the detection of small amount of polarisation. Further, since the vibration in the two emergent rays are perpendicular to one another, it is used for crossing two Nicols accurately.

**The Polaroid.** Nicol as a polariser, though very efficient, suffers from some serious disadvantages. A perfect crystal of calcite of large size is very rare and, therefore, very costly. With a small-sized Nicol, only a narrow beam of light can be transmitted ; further, the length of the Nicol being nearly three times its breadth, the field of view is very limited. Recently, a type of polariser called polaroid has been invented which has overcome some of the difficulties mentioned above It is made of fine crystals of herapathite which is per-iodide of quinine sulphate. It is a doubly refracting crystal and like tourmaline it absorbs the ordinary ray and transmits the extra-ordinary ray. Ultra-microscopic crystals of this substance are suspended in nitro-cellulose and forced under pressure through a horizontal slit. By this process all the tiny crystals are oriented in the same direction, thereby producing a large crystal of the same substance effectively. It is then mounted between two glass plates on which the direction of vibration of the transmitted beam is marked. By this process discs of polaroid up to six inches in diameter can be obtained at a cheap price.

The polaroid promises to have an important application in reducing the glare from polished surfaces, since such light is very often polarised. It can also be used in reducing the glare of head-lights in road traffic. For this purpose all the head-lights are to be fitted with discs of polaroids with vibration planes inclined at 45° to the vertical in the clock-wise direction when

viewed from the front of the head-lights. If the driver of a car A looks at the road illuminated by his own head light, through an eye-glass of polaroid similarly oriented, he will see the objects on the road in full illumination, because the two planes of vibration would be parallel. But if the driver of the car A looks at the head-light of another car B moving towards the car A from the opposite direction, then the vibration planes of the head-light covers of the car B and of the eye-glass of driver in A would be crossed, and therefore, the illumination due to car B would be cut off.

**Nicol as polariser and analyser.** A Nicol's prism can be used both for the production and detection of plane polarised light. Thus if two Nicols P and A are placed in the path of a beam of light, the beam will pass through both of them if the principal sections of the two Nicols are parallel to one another. But, if prisms are crossed, that is, if their principal sections are perpendicular to one another, no light will emerge through the second Nicol. In this case, the Nicol P is called the polarising Nicol or simply polariser, and the Nicol A is called the analysing Nicol or the analyser.

# CHAPTER VII

## INTERFERENCE OF POLARISED LIGHT

**44. Interference of Plane-polarised Light.** Polarised light can be reflected and refracted like ordinary light. A beam of heterogeneous plane-polarised light can also be dispersed into its constituent colours by refraction through a prism of glass. We shall discuss in this chapter the conditions of interference of polarised light.

Billet's split lens method of producing interference fringes is very convenient for this purpose. Light coming from an illuminated slit falls on two half lenses (Fig. VII-1) which converge the beams transmitted through them to two foci placed symmetrically about the axial line. Rays diverging from the two foci overlap in the region PQ where interference fringes can be formed.

**Experiment 1.** Place a Nicol in the path of the beam of light in front of the slit so that the two interfering beams may be plane-polarised. The interference fringes on the screen are not disturbed. Since, through a Nicol only extra-ordinary rays can pass, the experiment proves that *two extra-ordinary rays vibrating in two parallel planes can interfere like natural light.*

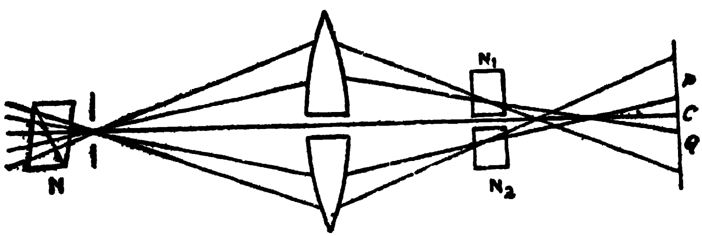



Fig. VII-1

**Experiment 2.** Place two calcite crystals $N_1$ and $N_2$ at the two foci with their principal sections parallel to one another and perpendicular to the principal section of the Nicol N. This arrangement does not disturb the interference fringes on the screen. Since the rays emerging from the crystals $N_1$ and $N_2$

vibrate in two parallel planes which are perpendicular to the principal sections of $N_2$ and $N_2$, the emergent rays must be ordinary. Hence, *two ordinary rays vibrating in parallel planes can interfere like natural light.*

**Experiment 3.** Let the principal section of the crystal $N_1$ be parallel to that of the Nicol N while the principal section of $N_2$ be perpendicular to that of the Nicol N. Under this circumstance the rays emerging from $N_1$ are extra-ordinary and those emerging from $N_2$ are ordinary. In this case all traces of interference fringes on the screen disappear.

Hence *two plane-polarised beams vibrating in two perpendicular planes cannot interfere destructively.*

**Experiment 4.** Let the principal sections of $N_1$ and $N_2$ be parallel to one an other and let the principal section of the Nicol N be inclined to that of $N_1$ or $N_2$ at an angle (say 45°). In this case the incident plane-polarised beam breaks up into two components by transmission through each of the crystals $N_1$ and $N_2$. The planes of vibration of two ordinary beams and also those of the two extra-ordinary beams are parallel. In this case instead of one system of fringes, two systems of superposed fringes are produced on the screen. One system of fringes is due to interference between two ordinary beams and the other is due to interference between two extra-ordinary beams but *no interference fringes are formed between the ordinary and the extra-ordinary beams* (as in Expt. 3)

**Experiment 5.** Let the principal sections of $N_1$ an $N_2$ be perpendicular to one an other and let the principal section of N be inclined to $N_1$ (or $N_2$) at an angle (say 45°). In this case the incident beam breaks up into two components by transmission through $N_1$ and $N_2$. But the plane of vibration of ordinary ray from N is parallel to the plane of vibration of the extra-ordinary ray from $N_2$ and *conversely.* Two systems of fringes are also produced in this case due to interference between ordinary and extra-ordinary rays whose planes of vibration are parallel. But they are displaced in opposite directions with respect to the line of symmetry. This is because ordinary ray travels more slowly in calcite than the extra-ordinary ray and, therefore,

on emergence from the calcite crystals there is introduced a relative difference of phase between the two interfering components. Hence *ordinary and extra-ordinary rays can interfere with one another destructively provided their planes of vibration are parallel.*

The results of the experiments described above are in agreement with the principle of superposition which requires that for destructive interference the vibration in the two interfering rays must be in the same direction. Two vibrations in perpendicular directions can also interfere with one another, but, instead of producing destructive interference they produce another type of polarisation — the elliptic polarisation which will form the subject-matter of this chapter. In the experiments above described the interfering rays are moving in the same direction. Hence absence of interference between two such coherent rays can only be explained on the assumption that vibrations in two rays are transverse and further the directions of vibration in the two interfering rays are mutually perpendicular to one an other.

**45. Phase-difference between two Polarised emergent rays from a crysttal.** We have seen that the lateral separation between the ordinary and the extra-

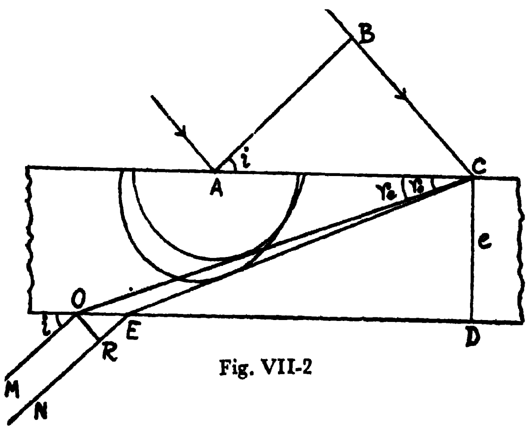



Fig. VII-2

ordinary rays obtained by double refraction through a crystal depends on the thickness of the crystal traversed by the rays. Again since the velocities of propagation of these rays

the crystal are different, (ordinary ray moving more slowly through negative crystal than the extra-ordinaryray) there would be a difference of phase between the rays on emergence. We now proceed to calculate this difference of phase between the two emergent rays.

In Fig. VII-2 let AB be the plane wave-front incident on a parallel slab of crystal of thickness $e$. Let CO and CE be the refracted wave-fronts within the crystal and OM and EN the emergent wave-fronts which are necessarily parallel to the incident wave-front. The effect of the plate is evidently to retard one wave-front over the other by the distance OR which is the normal separation between the two emergent wave-fronts.

Let the angle of incidence on the upper face of the crystal be $i$ and the angles of refraction for ordinary and extra-ordinary rays within the crystal be $r_0$ and $r_e$. From the geometry of the figure,

$$\text{OR} = \text{OE} . \sin i$$

$$= (\text{OD}-\text{ED}) \sin i$$

Now, $\text{OD} = e \cot r_0$

and $\text{ED} = e . \cot r_e$

Hence $\text{OR} = e(\cot r_0 - \cot r_e) . \sin i$

$$= e\left(\frac{\cos r_0}{\sin r_0} - \frac{\cos r_e}{\sin r_e}\right) \sin i$$

$$= e\left(\frac{\sin i}{\sin r_0} \cos r_0 - \frac{\sin i}{\sin r_e} \cos r_e\right)$$

or $$\text{OR} = l = e(n_0 \cos r_0 + n_e \cos r_e) \quad \ldots \quad (45{\cdot}1)$$

where $n_0$ and $n_e$ are the ordinary and the extra-ordinary refractive indices respectively. Evidently, the phase-difference between the two emergent rays is

$$\delta = \frac{2\pi}{\lambda} . \text{OR} = \frac{2\pi e}{\lambda} (n_0 \cos r_0 - n_e \cos r_e) \quad \ldots \quad (45.2)$$

In the particular case when the optic axis is parallel to the refracting face of the crystal and the incidence is normal as in Figs. VII-3 and VII-4 both ordinary and the extra-ordinary rays move in the same direction normal to the refracting surface but with different velocities. In the negative crystal (Fig. VII-3) the velocity of the ordinary ray is less than that of the extra-ordinary and in the positive crystal (Fig. VII-4) the

velocity of the extra-ordinary ray is less than that of the ordinary ray. Accordingly, in these cases, $r_e = 0$, and $r_o = 0$. For positive crystal the refractive index in this direction is the maximum and for negative crystal it is the minimum.

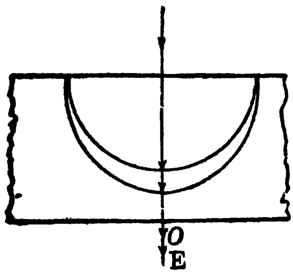



Fig. VII-3

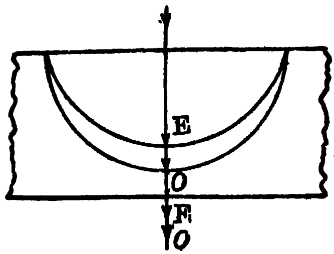



Fig. VII-4

Hence putting $n_e = n_E$ = the extra-ordinary refractive index in Eqn. (45·1), the path-difference between the ordinary and the extra-ordinary emergent rays is

$$l = e(n_E - n_o) \quad \ldots \quad (45\cdot3)$$

and the phase-difference is $\delta = \dfrac{2\pi e}{\lambda}(n_E - n_o)$ ... (45·4)

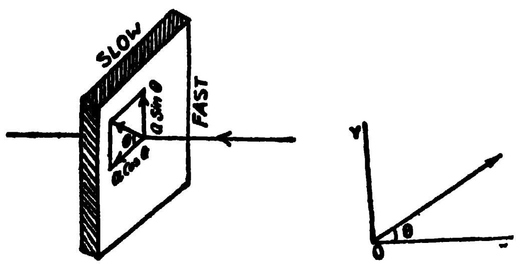



Fig. VII-5

Let a parallel beam of plane-polarised light be incident normally on a crystal plate cut with its face containing the optic axis. Let the plane of vibration of the incident beam make an angle $\theta$ with the optic axis (Fig. VII-5). If $a$ be the amplitude of vibration in the incident ray, the ray on entering the crystal breaks up into two components $a\cos\theta$ in the direction of the optic axis and $a\sin\theta$ in the direction perpendicular to the optic

axis. Since extra-ordinary ray vibrates in the principal section of the crystal and since inside a negative crystal the extra-ordinary ray moves faster than the ordinary ray, it is customary to call the direction of optic axis in a negative crystal, the **fast axis** and the direction perpendicular to the optic axis the **slow axis.** For obvious reasons, the optic axis in a positive crystal is called the 'slow axis' while the perpendicular direction is called the 'fast axis'.

**The quater wave-plate.** A quarter-wave plate is a crystal plate cut parallel to the optic axis and is of such thickness that when a ray of light is incident normally on the plate a path-difference of a quarter of a wave-length (or phase-difference of 90°) is introduced between the two emergent ordinary and extra-ordinary rays. The thickness of the plate required for this purpose can be calculated from Eqn. (45.3) by putting $l=\lambda/4$. Since, by Eqn. (45.3), the thickness $e$ of the crystal plate required depends on $\lambda$, the thickness of the quarter-wave plate would be different for different wave-lengths.

With sodium light ($5893\times10^{-8}$ cm), the refractive indices of calcite are $n_o=1{\cdot}65836$ and $n=1{\cdot}48641$. Hence the thickness of the plate of calcite crystal plate is to make a quarter-wave required.

$$e=\frac{5893\times10^{-8}}{4}(1{\cdot}65836-1{\cdot}48641)=253{\cdot}4\times10^{-8}\ \text{cm}.$$

With quartz plate illuminated by sodium light $n_o=1{\cdot}54425$ and $n_e=1{\cdot}55336$.

Hence a quarter wave-plate of quartz should have thickness

$$e=\frac{5893\times10^{-8}}{4}(1{\cdot}55336-1{\cdot}54425)=13{\cdot}6\times10^{-8}\ \text{cm}.$$

Quartz has no natural cleavage plane and hence to produce a quarter-wave plate of quartz, the crystal is to be cut and the cut faces are to be polished to optical flatness.

Mica and selenite are most commonly used for the construction of quarter-wave plates. Since mica can be easily split into very thin sheets, mica plates are specially used for this purpose. Mica is a biaxial crystal, the angle between the axes being any

thing between 0° and 42° depending on its chemical composition. The plane of cleavage of mica is perpendicular to the bisector of the angle between the two optic axes. Two of the three refractive indices of mica are nearly the same. For the construction of the quarter-wave plate clear and colourless specimen of mica with angle between optic axes as nearly equal to 0° as possible is chosen. For such mica the wave surfaces approximate to those of a positive uniaxial crystal.

**The half-wave plate.** For the purpose of optical testing half-wave plates are also used. The half-wave plate is similar to a quarter-wave plate in every detail, the only point of difference being that it introduces a path-difference of half a wave-length (or phase difference of 180°) between the two emergent ordinary and extra-ordinary rays. Hence the thickness of a half-wave plate for a particular wave-length must be double the thickness of a quarter-wave plate. If a beam of plane-polarised light is incident on a half-wave plate with its plane of vibration making an angle $\theta$ with the optic axis of the plate then the emergent light is also plane-polarised but its plane of polarisation makes an angle $(-\theta)$ with the optic axis.

**46. Elliptically and Circularly polarised Light.** Since the ordinary and the extra-ordinary rays move through the crystal in Fig. VII-5 in the same direction without separation, the two perpendicular vibrations $a \cos\theta$ and $a \sin\theta$ are superposed, the difference of phase between the two vibrations depending on the thickness of the crystal traversed by the rays (by Eqn. 45·4). If the vibration along the slow axis is represented by

$$x = a \cos\theta . \cos \omega t \qquad \ldots \qquad \ldots (46{\cdot}1)$$

that along the fast axis would be represented by

$$y = a \sin \theta . \cos (\omega t + \delta) \qquad \ldots \qquad \ldots (46{\cdot}2)$$

where $\delta$ depends on the thickness $e$ of the crystal traversed by the ray (Eqn. 45·4). The vibration curve in this case is obtained by eliminating $\omega t$ between the equations (46.1) and (46.2) thus :

From Eqn. (46·1),

$$\cos \omega t = \frac{x}{a \cos \theta}, \text{ so that, } \sin \omega t = \sqrt{1 - \frac{x^2}{a^2 \cos^2 \theta}}$$

Expanding (46·2) and substituting for cos $\omega t$ and sin $\omega t$, we get

$$\frac{y}{a \sin \theta}=\frac{x}{a \cos \theta} \cos \delta-\sqrt{1-\frac{x^2}{a^2 \cos^2\theta}}.\sin \delta.$$

or,
$$\left(\frac{y}{a \sin \theta}-\frac{x \cos \delta}{a \cos \theta}\right)^2=\left(1-\frac{x^2}{a^2 \cos^2\theta}\right) \sin^2 \delta.$$

or,
$$\frac{y^2}{a^2.\sin^2\theta}+\frac{x^2}{a^2 \cos^2\theta}-\frac{2xy.\cos \delta}{a^2.\sin \theta \cos \theta}=\sin^2\delta. \quad ...(46·3)$$

Eqn. (46.3) is the general equation of an ellipse of semi-axes equal to $a$ sin$\theta$ and $a$ cos$\theta$. The amplitudes of the two vibrations depend on the angle $\theta$ and the phase-difference $\delta$ depends on the thickness of the crystal plate.

**Particular cases. Case I.** If $\delta=2s\pi$ ($s=0, 1, 2, 3...$), Eqn. (46.3) transforms into

$$\frac{y}{a \sin \theta}-\frac{x}{a \cos \theta}=0$$

or
$$y=x \tan \theta \quad ... (46.4)$$

Hence the vibration-curve is a straight line. The emergent light is plane-polarised, the plane of vibration being inclined at an angle $\theta$ to the X-axis or the slow axis, that is, the plane vibration is the same as in the incident beam.

**Case II.** If $\delta=(2s+1)\pi$ ($s=0, 1, 2, 3...$), Eqn. (46·3) transforms into

$$\frac{y}{a \sin \theta}+\frac{x}{a \cos \theta}=0 \quad ... (46·5)$$

$$y=-x \tan \theta=x \tan (-\theta)$$

The light is again plane-polarised, the plane of vibration making an angle $(-\theta)$ with X-axis, that is, making angle $2\theta$ with the direction of incident vibration.

**Case III.** If $\delta=(2s+1).\pi/2$, Eqn. 46·3 takes the form

$$\frac{y^2}{a^2 \sin^2\theta}+\frac{x^2}{a^2\cos^2\theta}=1 \quad ... (46·6)$$

Hence the vibration curve is an ellipse, the axes of the ellipse being parallel and perpendicular to the principal section of the crystal.

If $\delta=\pi/2$, the component vibrations are $x=a\cos\theta\cos\omega t$ and $y=a\sin\theta.\sin\omega t$. But if $\delta=3\pi/2$, the two components are $x=a\cos\theta.\cos\omega t$, and $y=-a\sin\theta.\sin\omega t=a\sin\theta\sin(-\omega t)$. Hence, if the rotation in the former case is clock-wise, that in the latter case is anticlockwise, $\omega$ being of opposite sign.

Since the major and the minor axes of the ellipse in Eqn. (46·6) are respectively $a\cos\theta$ and $a\sin\theta$, *the ratio of the minor to major axes of the ellipse is equal to* $\tan\theta$. The amplitudes of $x$ and $y$ components of the two vibrations are $a\cos\theta$ and $a\sin\theta$. The resultant intensity of the elliptic vibration is equal to the sum of intensities of the two components and is equal to $a^2$ that is, equal to the intensity of the incident polarised beam.

**Case IV.** If in Eqn. (46·6) we put $\theta=45°$, we get $\sin\theta=\cos\theta=\frac{1}{\sqrt{2}}$ and it reduces to

$$x^2+y^2=\frac{a^2}{2}$$

so that the vibration curve is a circle of radius $a/\sqrt{2}$.

Hence the vibration curve changes from straight line to ellipse and from ellipse to straight line at different depths within the crystal and the cycle of changes repeats itself after equal thicknesses of the crystal. Fig. VII-6 shows the cycle of changes at different depths inside the crystal. The upper figure VII-6 gives the cycle for $\theta=45°$ while the lower figure shows the cycle for values of $\theta$ greater than 15°.


Fig. VII-6

**Plane, Circularly and Elliptically polarised light.** We have proved that light vibrations are transverse. When the transverse vibrations are executed in a fixed plane containing the ray the light is said to be plane-polarised. If the transverse vibration takes place in elliptic orbit round the direction of

the ray as axis, then the light is said to be elliptically polarised. If the transverse vibration takes place in circular orbits round the ray as axis, then the light is said to be circularly polarised.

In traversing a crystal cut parallel to the optic axis, a ray of light incident normally on the crystal changes its state of polarisation continuously from plane to elliptic, elliptic to circular (if, of course, $\theta=45°$) again to elliptic and then to the plane-polarised state and this cycle of changes is repeated at regular intervals of space.

**Condition of production of circularly-polarised light.** From what has been said above, a beam of plane-polarised light emerges as circularly polarised light when the beam is incident normally on a quarter-wave plate cut parallel to the optic axis with its plane of vibration inclined at angle of 45° with the optic axis. This introduces a phase-difference of an odd multiple of 90° (or of path-difference $\lambda/4$) between the ordinary and the extra-ordinary components.

**Analysis of Circularly and Elliptically Polarised Light. Circularly and elliptically polarised light viewed through a Nicol.** Circularly polarised light is produced when two components of light vibrations at right angles to one another and of the same amplitude are superposed the difference of phase between the two components being 90°. Since a circle is symmertical about its centre, any two perpendicular directions meeting at the centre of the circle may be taken as the two perpendicular axes of vibration. When a beam of circularly polarised light passes through a Nicol, it immediately breaks up into two components vibrating parallel and perpendicular to the principal sections of the Nicol. The component vibrating perpendicularly to the principal section is totally reflected and we get only one polarised beam emerging from the Nicol. Hence the transmitted light contains one of the perpendicular components which is the same for all orientations of the Nicol. Thus, by rotating the Nicol placed in the path of circularly polarised light, the intensity of the transmitted light does not change. In this respect circularly polarised light behaves like natural light.

Elliptically polarised light is produced by superposition of two rectangular vibrations of different amplitudes and phases. When elliptically polarised light is viewed through a Nicol, the intensity of the transmitted light changes from maximum to minimum values twice in each revolution of the Nicol but the minimum is never zero. The intensity is maximum when the principal section of the Nicol is parallel to the major axis and it is minimum when the principal section of the Nicol is parallel to the minor axis. This is also the characteristic of partially polarised light.

It follows that an analysing Nicol alone is not sufficient for detection of circular and elliptic polarisation. The methods of analysis of circularly and elliptically polarised light are described below :

**The apparatus.** For the purpose of such analysis an ordinary spectrometer with some attachments may be conveniently used. The attachments consist of three graduated circles $S_1S_2S_3$ fixed to three short tubes with their planes perpendicular to the axes of the tubes. Two of these tubes $T_1$ and $T_2$ slip over the object glasses of the telescope and the collimator respectively. Inside the tube over the collimator a Wollaston's prism W or a polarising Nicol may be placed. The tube over the object-glass of the telescope contains the analysing Nicol N. The third circle is fixed to another tube $T_3$ which is mounted on

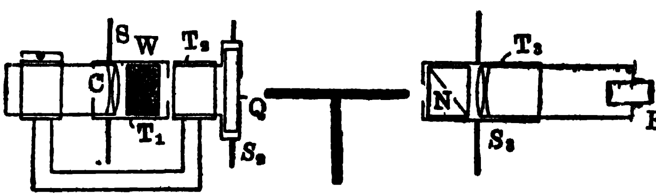



Fig. VII-7

the barrel of the collimator tube by means of a rigid framework. Inside this tube a quarter-wave plate Q is placed. By means of three fixed indices the angles of rotation of the circles can be measured.

**Adjustment of the apparatus.** Make the slit of the collimator vertical by means of a plumb line. Level the spectrometer and adjust it for parallel rays in the usual way and place the collimator and the telescope in a line. Then place an Wollaston's prism inside the tube $T_1$ and rotate the tube $T_1$ till the two images of the slit partly overlap in a vertical line. The vibration of light forming the two images will now be vertical and horizontal. Now place the analysing Nicol N inside the tube $T_1$ with its shorter diagonal placed approximately horizontal and fix the index of the graduated circle mounted on $T_2$ at zero. Rotate the Nicol slightly to one side or the other till one of the images of the slit is extinguished. The principal section of Nicol N is then horizontal. Then take the reading of the index on $T_2$. Next replace the Wollaston's prism by a polarising Nicol with its shorter diagonal placed approximately vertical. Fix the index of the graduated circle attached to $T_1$ at zero. Rotate the Nicol P slightly till the image of the slit is extinguished and read the position of the index.The principal section of the polarising Nicol P is now vertical. Now place a quarter wave-plate inside the tube $T_3$ and illuminate the slit of the collimator with light to be used. On looking through the telescope the field will appear to be bright, because the transmitted light through the quarter-wave plate is elliptically polarised. Rotate the quarter-wave plate till the field of view is again completely dark. This will happen only when the 'fast and 'slow' directions are parallel to the principal sections of the polarising and analysing Nicol. By this process the 'fast' and 'slow' directions of the quarter-wave plate are made vertical and horizontal. The reading of the index mark on the scale attached to tube $T_3$ is now taken. The instrument is now ready for use.

**Analysis of elliptically polarised light.** The analysis of elliptically polarised light involves the determination of the (*a*) positions of the major and minor axes of the ellipse (*b*) the difference of phase between vibrations executed along the major and minor axes and (*c*) the ratio of the major to the minor axes. The considerations underlying the analysis of elliptically polarised light are

(1) the analysing Nicol cannot extinguish elliptically polarised light. For extinction by a Nicol the light must be plane-polarised.

(2) Hence for extinction the elliptically polarised light must first be converted into plane-polarised light. The method of conversion depends on the following facts :

In the case of elliptically polarised light the difference of phase between the perpendicular components vibrating along the major and minor axes of the ellipse is $\delta=(2s+1)\frac{\pi}{2}$ where $s$ is on integer (Case III. P. 215). This can be converted into linear vibration in a plane if $\delta$ is increased or diminished by $\pi/2$ so that $\delta$ becomes $(s+1)\ \pi$ or simplo $s\pi$ where $s$ may be an odd or even integer (See cases I and II. P. 215). This difference of phase can be introduced by placing a $\lambda/4$-plate in the path of the elliptically polarised light. It is to be noted that this additional phase-difference is to be introduced between vibrations excuted along the major and minor axes of the elliptic vibrations to be analvsed. Hence the principal and normal sections of the $\lambda/4$ plate must be rotated so as to be parallel to the major and minor axes of the elliptic vibration. It is only under this condition that the emergent rays can be extinguished by the analysing Nicol. Hence in the position of extinction, the principal and normal sections of the $\lambda/4$ plate are parallel to the major and minor axes of the ellipse or vice versa.

When two rectangular vibrations are compounded into a rectilinear vibration by introducing a suitable phase-difference

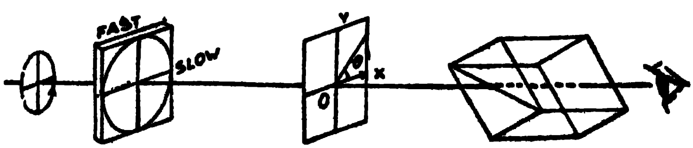



Fig. VII-8

$(\delta=s\pi)$ between the components, the tangent of the angle which the resultant vibration makes with the direction of X-component is equal to the ratio of amplitude of the Y-component to that of

the X-component (Case III 215). If the principal and normal sections of the quarter wave plate are parallel to the major and minor axes of elliptic vibration, then the emergent light is plane-polarised, the direction of the emergent light vibration making an angle $\theta$ with the 'slow' axis of the $\lambda/4$ plate as shown in Fig. VII-8. Hence the principal section of the analysing Nicol must make an angle $(90+\theta)$ with the direction of the 'slow' axis for extinction. The tangent of this angle $\theta$ is equal to the ratio of the major and minor axes of the elliptic vibration which must be equal to the ratio of amplitudes of the rectangular vibrations.

**Analysis with quarter wave plate.** Hence to analyse elliptic vibration with a quarter-wave plate, we set the apparatus as described under Fig. VII-7. For the laboratory purposes a beam of elliptically polarised light can be obtained by fixing with wax a thin piece of mica over the end of the polariser. This restores light in the field of view of the telescope showing that light transmitted through mica is elliptically polarised.

Then rotate the $\lambda/4$-plate through small angles in steps of about 5 degrees and set the analyser every time for minmum intensity. Repeat this process till complete extinction occurs. The positions of the 'slow' and 'fast' axes of the $\lambda/4$-plate then gives the positions of the major and minor axes of the ellipse. Let $a$ and $\beta$ be the total angles through which the analyser and the quarter-wave plate have been rotated from their respective positions before the mica sheet was introduced. Then the ratio of the major and minor axes is given by tan $(\alpha-\beta)$. This idea will be clear from Fig. VII-9 where OV and OH represent the vertical and horizontal lines to which the principal sections of analyser and polariser were made parallel during adjustment. From the method of adjustment given above these are also the directions of the 'slow' and 'fast' axes of the quarter-wave plate. Let the ellipse shown in Fig. VII-9 represent the path of the particle in the elliptically polarised light which is to be analysed. For complete extinction, the 'slow' and 'fast' axes

must be rotated through angle $\beta$ into positions OX and OY so as to coincide with the major and minor axes of the ellipse. The phase-difference introduced by the $\lambda/4$-plate converts elliptic to rectilinear vibration in the direction OR which makes an

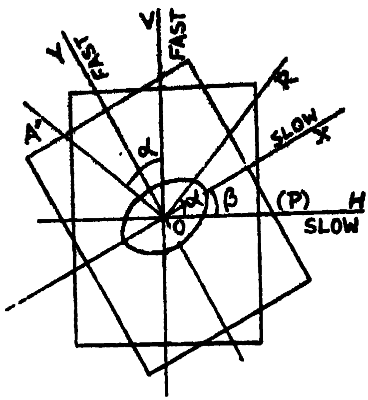



Fig. VII-9

angle $\alpha$ with OH. Hence the angle through which the analyser must be rotated for extinction is $\alpha$. The angle which the resultant OR makes with the 'slow' axis in the new position is evidently $(\alpha-\beta)$.

**Direction of rotation in circularly polarised light.** Circularly polarised light is composed of two rectangular vibrations of the same amplitude and of difference of phase equal to $(2s+1)\pi/2$. For the sake of definiteness let us suppose that the ray of light is moving towards the reader and the X- and the Y-components are respectively in the horizontal and vertical directions with a phase-difference of 90° between them. Let the X-component be 90° ahead in phase of the Y-component. This means that the displacement in the X-vibration is maximum in the positive direction at the instant when the Y-vibration is just producing a displacement, through the mean position in the positive direction (Fig. VII-10) Accordingly, the rotation in the circle will be in the anti-clockwise direction. If the Y-component is 90° ahead in phase, then displacement due to Y-vibration would be maximum in the

positive direction at the instant when the X-vibration is just causing a displacement in the positive direction through the origin (Fig.VII-11). Hence the rotation is clockwise or

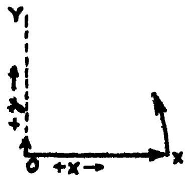



Fig. VII-10

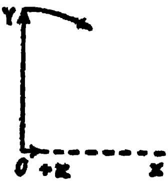



Fig. VII-11

anti-clockwise according as the Y-vibration or X-vibration is 90° ahead in phase as compared to the X-vibration or the Y-vibration respectively.

In the case of negative uniaxial crystal the ordinary ray moves more slowly than the extra-ordinary ray. Hence in the case of circularly polarised light produced by transmission through negative crystal, the extra-ordinary ray is 90° ahead in phase than the ordinary. With positive crystals where ordinary ray moves faster than the extra-ordinary ray, the ordinary ray in the circularly polarised light is 90° ahead in phase with respect to the extra-ordinary ray.

A quarter-wave plate can be used for detection of direction of rotation in circularly polarised light. Let the rotation in the incident beam be in the clock-wise direction (Fig. VII-12), so that the Y-component is 90° ahead of the X-component, the X-and

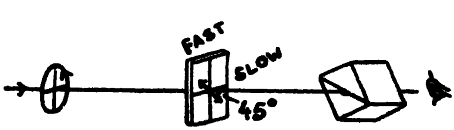



Fig. VII-12

the Y-components being taken to be parallel to the 'slow' and 'fest' axes of a quarter-wave plate placed perpendicularly to the path of the rays. On entering the $\lambda/4$ plate, the Y-component

will move faster through the plate than the X-component. because Y-axis is the faster axis through the $\lambda/4$ plate. Hence, on emergence from the plate the Y-component would be ahead of this X-components by $180^0$. Hence the resultant vibration would be rectilinear inclined at angle of $-45°$ to the fast axis, since the amplitudes of the two components are equal. For extinction the analysing Nicol must make an angle of $+45°$ with the slow or X-axis.

If the rotation in the incident beam is in the anti-clockwise direction as in Fig. VII-13, then the X-component would be $90°$ ahead of the Y-component. Through the quarter wave plate the Y-component would move more faster than the X-component.

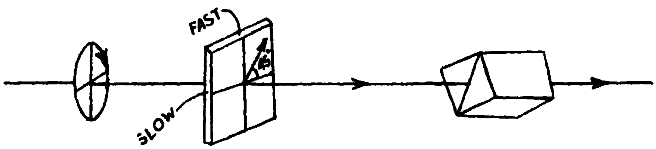



Fig. VII-13

Hence on emergence from the $\lambda/4$-plate the Y-component be $90°$ ahead of the X-component, so that the total phase-difference between the X- and the Y-components on emergence would be zero. The resultant vibration would again be rectilinear but its direction would make an angle of $+45°$ with the 'slow' axis. For extinction the principal section of the analysing Nicol must make an angle of $-45°$ with the slow or the X-axis.

*Thus rotation is clock-wise or anticlock-wise according as the principal section of the analysing Nicol makes an $+45°$ or $-45°$ with the axis of the quarter-wave plate designated 'slow'.*

# CHAPTER VIII

## ROTATORY POLARISATION

**47. Rotating Crystals** We have seen that when light is transmitted through a crystal in a direction parallel to the optic axis, there is no double refraction. It was shown by Arago in 1811, that there are exceptions to this general rule. If a plate of quartz cut perpendicularly to the optic axis is placed between two crossed Nicols illuminated by sodium light, the field viewed through the analyser becomes illuminated. On turning the Nicol through a certain angle, the field again becomes dark. This means that the light emerging from the crystal is plane-polarised and that the plane of polarisation is rotated through a certain angle which is equal to the angle through which the analysing Nicol has been turned for extinction. Such crystals are said to be optically active or rotating crystals. In this respect the quartz crystal can be divided into two groups:—(1) those which turn the plane of vibration to the right when looked against the direction in which light is moving. Such crystals are called right-handed or dextrogyrate; (2) those crystals which rotate the plane of vibration to the left when looked against the direction in which light is moving are called left-handed or laevogyrate.

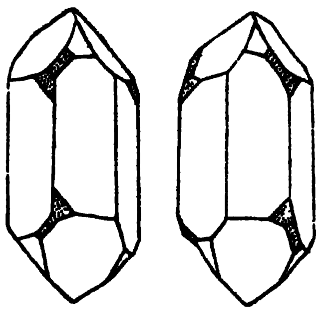


Fig. VIII-1

Experiment shows that the amount of rotation is propotional to the thickness of the crystal plate traversed by the ray and also on the wave-length of light used. It is found if the crystal is turned through 180° so that light traverses the crystal in opposite direction, then the sense of rotation of the plane of vibrations is not reversed.

In many cases the rotating crystals can be recognised by visual examination of the shape of the crystal. On examining the rotating quartz crystal in Fig. VIII-1, it will be found that some of the angles at the junctions of the prism and the pyramid are absent and are replaced by facets or small secondary planes (shown shades in the diagram). These secondary planes are obliquely inclined to the other faces of the crystal. In the same crystal all the secondary planes lean in the same direction. The crystal produces a right-handed rotation if the planes lean to the

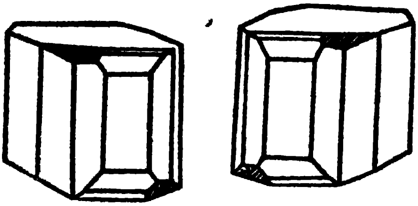


Fig. VIII-2

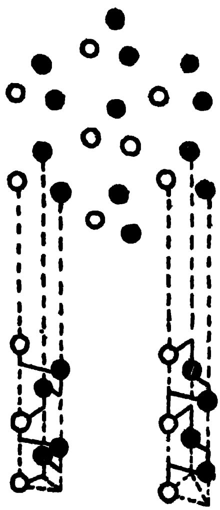


Fig- VIII-3

right when the crystal is viewed with the apex of the pyramid uppermost. If the secondary planes lean to the left, the rotation is left-handed. Such crystals are called enantiomorphcıs (*enantios*, opposite; *mor: phir*, form). For the same substance such crystals usually occur in two varieties, one of which is the mirror image of the other producing rotation of the plane of vibration in opposite directions.

With regard to the internal structure of these crystals, it has been found that all of them possess one or more screw axis of symmetry. This means that the atoms or molecules constituting the crystal are arranged on a spiral as shown in Fig. VIII-3. If the spiral is right handed, then the rotation is also right-handed. Similarly, a left-handed spiral produces a left-handed rotation.

The optical activity is not peculiar to quartz alone. Many liquids and vapours also possess the power of rotating the plane of vibration. Optically active substances can be divided into two

classes. In class (1) we group these substances which retain their activity in the crystalline state as well as in the fused or dissolved state. In class (2) we include those substances which are active in the crystalline state only.

Naturally, substances belonging to class (1) are optically active in their molecular state. Some of the organic compounds, such as, lactic acid, tartaric acid, many of the sugars, and some inorganic co-ordination compounds belong to this class. It is found that tetravalent carbon atom attached asymmetrically to four different groups or atoms forming a tetrahedron structure with the carbon atom occupying the central position usually give two different structures of the molecule, one of which is the mirror image of the other. These molecules possess the power of optical rotation, one of these structures being laevogyrate and the other dextrogyrate. In no case, an exception to this rule has been found. Such molecules form spiral arrangement in the crystalline state. Naturally, the activity of crystals of such substances is the resultant of activity of the molecules and that due to the spiral arrangement. These two effects usually reinforce one another, the activity due to the spiral arrangement being much greater in most cases. The optical activity of substances belonging to class (2) is due to spiral arrangement of the molecules in the crystal, so that the substance loses its activity when the structure is lost by fusion or solution.

It would appear at first sight that a substance belonging to class (1) would lose its activity when it is dissolved in a solvent, since the random orientation of the molecules in the solution would cancel out each other's effect. But a right-handed screw is always right-handed from whichever end it is viewed. Hence, no matter how the molecules are oriented, the total rotation is the sum of rotations produced by the molecules lying in the path of the beam of light traversing the solution. It is for the same reason that the sense of rotation produed by a crystal is not changed if the crystal is turned through 180."

**48. Rotatory Dispersion.** Both theory and experiment point to the conclusion that the amount of rotation produced by an active-crystal of given thickness depends on the

wave-length of light used. The following table gives the rotation produced by a plate of $\alpha$- quartz of one millimetre thickness cut perpendicularly to the optic axis for solar line of different wave-lengths.

| Solar line | wave-length (A°) | Rotation |
|---|---|---|
| B | 6870 (Red) | 15°45′ |
| D | 5893 (orange) | 21°43′ |
| F | 4861 (Blue) | 32°46′ |
| H | 3968 (violet) | 51°11′ |

If a plate of active crystal illuminated by white light is inserted between two crossed Nicols the light emerging from the analysing Nicol would be brilliantly coloured. This will be clear by referring to Fig. VIII-4 where OE and ON are the directions of principal section of the analyser and the polariser respectively. After transmission through the crystal plate, the planes of vibration of the different colours are rotated by different amounts and occupy the positions represented by OR, OY, OG. OB etc. The analysing Nicol transmits only the components $OE_R$ $OE_Y$, $OE_G$, $OE_B$ etc. These vibrations mingled up together produce a coloured beam of light. It is evident that the colour of the transmitted beam would alter if the analysing Nicol is rotated about its axis.

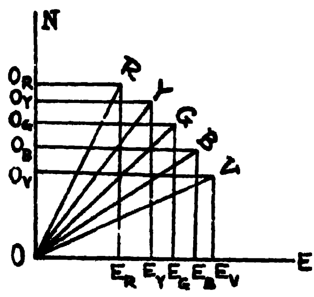


Fig. VIII-4

**Tint of passage.** With a plate of quartz of thickness less than 5 mm. there is a particular position of the analysing Nicol for which the emergent beam possesses a greyish-violet colour which is called *the tint of passage*. This happens when the principal section of the analyser is perpendicular to the plane of vibration of greenish-yellow rays, which is the most intense part of the visible spectrum. This part of the spectrum being quenched by the analysing Nicol, the illumination of the field is

minimum. The colour of the emergent beam is necessarily produced by the combination of the less intense violet and red ends of the spectrum. If the analysing Nicol is rotated slightly to one side from this position the field becomes red. If the Nicol is turned slightly to the other side, the field appears blue. The transition from red to blue being very critical, the position for the tint of passage can be very accurately determined.

**49. Polarimeters:** Polarimeters are instruments designed for accurate determination of angle of rotation of the plane of vibration produced by an active substance placed between analysing and polarising Nicols. As already mentioned when an optically active substance is placed between two crossed Nicols, the field of view becomes illuminated and to extinguish the light the analysing Nicol is to be rotated through a certain angle which is the measure of the rotation produced. This simple method of measuring the rotation does not lead to very accurate results, since the field remains dark over an appreciable angle near about the position of extinction. For this reason various arrangements have been devised for accurate setting of the analysing Nicol. This requires some auxiliary pieces of apparatus, which instead of making the whole field dark divides the field of view into two or three equal parts. By turning the analysing Nicol the intensity over the different parts of the field of view can be adjusted to exact equality, since the eye is very sensitive to the contrast in brightness over two adjancent illuminated surfaces. We describe below some of these devices which are generally used for determination of optical rotation.

**Laurent's polarimeter.** The arrangement for measuring the rotation known as Laurent's polarimeter is shown in Fig. VIII-5.

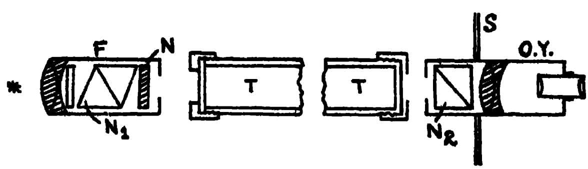



Fig. VIII-5

Light coming from a source S is rendered parallel by the converging lens L. It then falls on a light filter and is polarised

by the Nicol $N_1$. By means of the half-shade plate N the field is divided into two equal halves whose brightness is adjusted to exact equality by the analysing Nicol $N_2$ which receives the beam emerging from the rotating liquid contained in the tube T, the two faces of which are closed by plane parallel glass plates. OT is a short-focus observing telescope which is focussed on the half-shade. The analysing Nicol $N_2$ carries a circular graduated scale which rotates with the Nicol and the angle through which the Nicol is turned is read with the help of a fixed vernier.

**The half-shade** is a circular plate one half of which is of quartz cut parallel to the optic axis and the other half of which is of glass. The thickness of the quartz plate is such that it introduces a phase-difference $\lambda/2$ between the two emergent ordinary and extra-ordinary rays. The thickness of the plate glass is so adjusted that the absorption of light by the plate glass is the same as that abscrbed by the quartz plate.

Let ROS (Fig. VIII-6) be the direction of the optic axis of the quartz plate and OP the direction of vibration of the incident polarised beam making an angle $\theta$ with the optic axis. By transmission through the glass plate, the plane of vibration is not altered. On emergence from the half-wave plate of quartz, the plane of vibration is rotated through angle $2\theta$ into the position OQ. If the principal section of the analysing Nicol is parallel or perpendicular to ROS, then equal components of the two vibrations OP and OQ would be transmitted through the Nicol and both halves of the field of view would appear equally bright. In any other position of the Nicol, the brightness of the two halves would be unequal. For a particular minimum value of the angle POQ, the instrument is very sensitive. This position of maximum sensitivity can be determined by rotating the polarising Nicol $N_1$ by means of

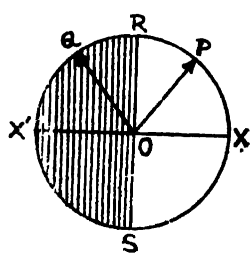



Fig. VIII-6

a handle provided with the instrument. The thickness of the half-shade depends, of course, on the wave-length of light used.

**The biquartz polarimeter.** This instrument is similar to Laurent's polarimeter, the only modification being that the half-shade is here replaced by a biquartz plate. This consists of two semi-circular plates of quartz of equal thickness (about 4 millimetres) cut perpendicularly to the optic axis (Fig. VIII-6). One of these plates rotates the plane of vibration to the right and the other to the left. If a beam of white light is transmitted through the semi-circular plates each will produce rotatory dispersion; that is, the different coloured constituents are rotated by different amounts. The thickness of the quartz plates in the biquartz polarimeter is so adjusted that the greenish-yellow part of the spectrum is rotated through 90°. Accordingly, if the principal section of the analysing Nicol is placed parallel to the plane of vibration of the incident light, the yellow greenish part of the spectrum is cut off and the emergent light shows the tint of passage. On rotating the analysing Nicol slightly to one side, one half of the field is red and the other half is blue. The transition from red to blue being very critical, the setting of the analysing Nicol can be effected with great accuracy.

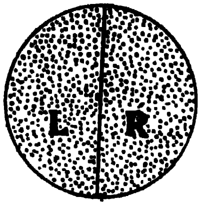


Fig. VIII-6

**Lippish two-prism polarimeter.** In this arrangement, instead of one polarising Nicol, two prisms N, $N_1$ (Fig. VIII-7)

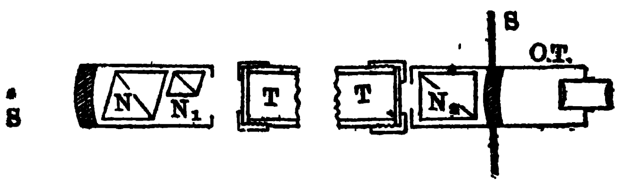


Fig. VIII-7

are used. The aperture of $N_1$ is half that of N, so that it covers half the field of view. The principal sections of N and $N_1$ are

slightly inclined to one another. Hence the planes of vibration of light incident on T, the rotaing substance, are slightly inclined to one another as shown in Fig. VIII-8. In the absence of the rotating substance, the two halves of the field would be equally bright, if the principal section of the analysing Nicol bisects the angle between the two directions of vibration OP and OQ. When the rotating substance is placed between the analysing and the polarising Nicols the planes of vibration in the two halves of the field are rotated by the same amount in the same direction. Hence, for extinction, the analysing Nicol must also be rotated by the same amount.

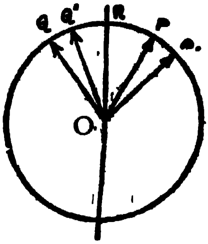



Fig. VIII-8

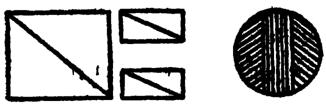


Fig. VIII-9*a*

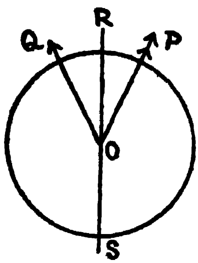



Fig. VIII-9*b*

**Lippish three-prism polarimeter.** To facilitate judgment of equality of brightness, modern type of Lippish polarimeter uses two auxiliary polarsing Nicols, so that the field of view is divided into three equal parts (Fig. VIII-9*a*). The principal sections of the auxiliary Nicols are parallel to one another and slightly inclined to that of the polarising Nicol so that planes of vibration of two outer parts of the field are parallel to OP (Fig. VIII-9*b*) and that in the central part is parallel to OQ. When the principal section of the analysing Nicol is parallel to ROS which bisects the angle POQ, the three parts of the field are equally bright.

**50. Specific and Molecular Rotations.** It is found experimentally that

(1) the rotation produced by a substance at a given temperature and for a given wave-length of light used is proportional to the thickness of the substances traversed by light.

In the case of solution of an active substance in an inactive solvent,

(2) the rotation produced by a substance at a given temperature and for given wave-length of light used is proportional to the concentration of the solution.*

If $\theta$ represents rotation in a length $l$ of the solution traversed by light produced by a solution of concentration $c$, then

$$\theta \propto l \text{ when } c \text{ is constant}$$

and $\quad \theta \propto c$ when $l$ is constant

so that, $\quad \theta = s.\ l.\ c.$

where $s$ is tne constant of proportionality called specific rotation. It is evidently the rotation produced by a solution of unit length and unit concentration. For technical purposes the unit of length is taken to be one decimetre and the unit of concentration is taken to be one gramme of the active substance in one cubic centimetre of solution with an inactive solvent. The constant of proportionality $s$ depends on temperature, wave-length of light and also on the nature of the active substance. In the determination of specific rotation it is, therefore, necessary to work at a constant temperature and with definite wave-length. The constant temperature is chosen to be 20°C and the wave-length of light to be used is chosen to be 5893A (sodium light).

There is no definite law of variation of specific rotation with temperature. Thus, specific rotation of turpentine diminishes with rise of temperature while in the case of quartz it increases

*It is found that the solvent, however inactive it may be, exerts some influence on the activity of the substance dissolved. Thus, the specific rotation of pure turpentine is 37°·01, while its value when dissolved in inactive solvents, such as, alcohol, benzol, and acetic acid are respectively 36°·974, 36°·970 and 36°·894.

as the temperature rises. Regarding dependence of specific rotation on the wave-length, Biot found that it is approximately proportional to the inverse square of wave-length.

**Molecular rotatory power.** As already mentioned (see p. 228) substances belonging to one class retain their rotatory power in the solid, liquid, as well as in the vaporous states. This has led to the supposition that molecules of these substances are themselves optically active. Hence we also use the term molecular rotatory power which is defined to be the product of specific rotation and the molecular weight of the substance.

**51. Saccharimetry** One of the most important technical applications of polarimeters is for determination of strength of sugar solution and the determination of impurity in such solution. Polarimeters used for sugar analysis are called saccharimeters. Pure cane-sugar or sucrose ($C_{12}H_{22}O_{11}$) is dextrogyate. Its specific rotation is $+\ 66{\cdot}5°$ at 20°C with sodium light. The value of the specific rotation ($s$) being known, the concentration of the sugar solution can be determined from the rotation $\theta$ produced by a length $l$ of the solution filling the tube of the polarimeter. This method is called the *direct method.*

Solutions of sugar usually contain impurities which are optically active. To determine the strength of sugar solution in such cases the following method known as the invert method is used. By the process of chemical invertion* the dextro-rotatory sucrose is converted into laevo-rotatory products—glucose and fructose. Glucose is dextro-rotatory ($s=52°5$) and fructose is laevo-rotatory ($s=—92{\cdot}5°$).

To determine the concentration of sucrose under test a given solution of the substance (say 100 c.c.) is divided into two equal halves (of 50 c.c. each). The optical rotation for one half of the solution is determined by the direct method. If $c$ be the

*This consists of hydrolysis of sucrose with dilute $H_2SO_4$ or HCl according to reaction

| | | | |
|---|---|---|---|
| $C_{12}H_{22}O_{11}+$ | $H_2O=$ | $C_6H_{12}O_6$ | $+\ C_6H_{12}O_6$ |
| (Sucrose) | | (Glucose) | (Fructose) |
| 342 | 18 | 180 | 180 |

concentration of this half of the solution and $l$ the length of the polarimeter tube, then, rotation $\theta$ is given by

$$\theta_1 = slc_0 + \rho = 66{\cdot}5 \times lc + \rho \qquad \ldots \quad (50.1)$$

where $\rho$ is the rotation produced by the impurities in the solution.

The other half of the solution is then inverted* and diluted to 100 c.c By the process of inversion the concentration of the inverted sucrose is reduced in the proportion of 360/342. Since the solution is diluted to double its original volume, the concentration of the inverted sucrose solution is $\frac{c}{2} \times \frac{360}{342}$. Since the resultant specific rotation of glucose and fructose is $(52{\cdot}5 - 92{\cdot}5) = -40^\circ$, the rotation produced by inverted sucrose solution is

$$-\theta_2 = -\frac{c}{2} \times \frac{360}{342} \cdot l \times 40 + \rho \qquad \ldots \quad (50{\cdot}2)$$

Subtracting Eqn. (50.1) from Eqn. (50.2)

$$\theta_1 + \theta_2 = cl \left[ 66{\cdot}5 + \frac{180 \times 40}{342} \right] = 87{\cdot}55 \times cl.$$

so that

$$c = \frac{\theta_1 + \theta_2}{87{\cdot}55 \times l}$$

*By the process of inversion it is only the sucrose content in the solution that is inverted. The impurities in the solution remain unaffected.

# CHAPTER IX

## SPECTROSCOPY

**52. The Complete Spectrum.** It was observed by Newton that a beam of sunlight on being refracted through a prism is broken up into differently coloured rays which are deviated by different amounts. This coloured band of light which is formed of differently coloured images of the slit has been called the spectrum. This region of the spectrum is characterised by its capacity to produce physiological action on the optic nerves which is interpreted as vision. From the physical point of view each colour corresponding to any point in the spectral region is specified by a number which is called the frequency in order to correlate light with waves propagated in space supposed to be filled up by a hypothetical medium called the ether. In this sense light is etheral waves of different wave-lengths and frequencies. For obvious physical reasons, all the different wave-lengths are propagated with the same velocity in the free space. But their motion is impeded when the waves travel through space occupied by material media, different material media affecting the velocity of waves in different ways. We have seen that the change in velocity of light through a material medium is associated with the change in wave-length but not in frequency. The range of wave-lengths in the spectrum under consideration is from $0{\cdot}43\mu$* for the violet light to $0{\cdot}75\mu$ for the red light.

There is no reason why the range of wave-lengths that can be generated in and propagated through ether should end abruptly at $0{\cdot}43\mu$ and $0{\cdot}75\mu$. The fact is that ether can transmit waves of all possible lengths. Of these waves the range of wave-lengths lying between $0{\cdot}43\mu$ and $0{\cdot}75\mu$ can act upon the optic nerves and is, therefore, called the region of *visible spectrum.*

* $\mu$ is another unit of wave-length. It is called *micron* and is equal to $\frac{1}{1000}$ of a millimetre$=10^{-4}$ cm. Millimicron is $\mu \times 10^{-3} = 10^{-7}$ cm.

On either side of the visible region there extends a vast field of spectrum which is invisible and requires special instruments for detection.

**The infra-red spectrum.** In the year 1800, Sir John Herschel tried to find out with the help of a delicate thermometer the region of the spectrum which had the least heating effect. He found that the thermometer showed a steady rise of temperature as it was moved from the violet to the red and it continued to rise and attained its maximum value at a short distance beyond the red end of the spectrum. He resumed his work in 1840 and proved conclusively that there actually exists a region of the spectrum beyond the red which is characterised by its heating effect. He called this region of the spectrum the infra-red region. The wave-nature of the infra-red radiations has been established by proving that they can be reflected, refracted, dispersed, made to interfere and also be polarised. The wave-lengths of dark-heat rays have been measured up to $67\mu$ by Prof. Rubens and Prof. Nicols in 1897 and 1898. Recently various methods have been devised for studying the far infra-red region of the spectrum. These methods would be discussed in detail later on.

**Wireless or Hertzian waves.** Skipping over a region of longest dark-heat waves which yet remains to be explored, (with regard to their methods of production and detection) we come to a region of the spectrum corresponding to wave-length of the order of $4000\mu$ or 4 millimetres. These waves are generated in the ether by means of oscillations set up in an electrical circuit in the fashion first discovered by Hertz. Later on Professors Lodge, Rhigi, Bose, Trouton, Fleming and many, others have carried on extensive researches with these electrically generated ether waves. Hertz himself, worked with waves of length lying between 1 or 2 to 30, or 40 feet. More recently waves of length ranging from 800 to 1000ft. have been employed in wireless telegraphy.

**The ultra-violet region.** Soon after Herschel's discovery of the infra-red region of the spectrum, the question naturally arose whether there existed a region of the spectrum beyond the

violet end. The attention of the scientists was drawn to the observation of Scheel that a photographic plate was blackened when it was placed beyond the violet end of the visible spectrum. This region of the spectrum which is invisible to the eye and which can yet affect the photographic plate is called the ultra-violet region, the corresponding rays are called the *ultra violet rays* and also the *actinic rays*. In 1893, Schumann measured wave-lengths as short as $3 \cdot 1 \mu$.

**The X-Ray region.** In the year 1885 Rontgen made the remarkable discovery that a type of radiation is produced when a stream of fast moving electrons strikes a metal target. They are characterised by their high penetrating power and quick photo-chemical reaction. At first there was a great controversy regarding the nature of these radiations and hence they were called X-Rays or unknown rays. In 1899 Haga and Wind took X-ray photograph of diffraction pattern produced by a V-shaped slit which resembled the diffraction pattern of the same slit produced by visible light. In the year 1912 Laue conceived the idea of using a crystal as a three dimensional grating for diffraction of X-Rays and ever since a crystal is being used for this purpose. In 1906 Barkla proved that X-ray could be polarised by scattering in exactly the same way that ordinary light is polarised through scattering by small particles. This establishes the identity of X-Rays with ordinary light, the only difference being that wave-length of X-Rays is very short in comparison with that of the ordinary light.

**The Gamma-ray region of the spectrum.** These are the types of radiation emitted by a radio-active substance in the process of radio-active disintegration. Their penetrating power is much greater than that of the hardest X-ray known—even a 30-cm. shield of iron failing to screen away some of the radiations coming from radio-active substances. Evidently $\gamma$-rays are radiations of much shorter wave-length than those of X-rays, Rutherford and Andrade measured the wave-length of $\gamma$-rays by a method similar to that adopted for X-rays. The observed wave-length of $\gamma$-rays extends from 1323 to 16 X-ray unit (The X.U is equal to one thousandth part of an angstrom, that is, $10^{-11}$cm.)

**The cosmic rays.** In the year 1900, Elster, Geitel, and Wilson observed that a charged electroscope always showed a leakage, no matter how excellent the insulation might be. By shielding the electroscope in a metal casing, the rate of leakage was reduced. This proved that the leakage was due to some external influences. On taking the electroscope to a height above the earth's surface, the effect at first diminished, reaching a minimum value at a height of 2000 metres and then increased steadily with height. This has led to the supposition that some type of radiations are coming to the earth from outside whose intensity according to Hess was the same during day and night. These rays are characterised by their highest penetrating power. They can pass through 280 metres of water. These rays are called the cosmic rays. If they are regarded as light rays, their wave-length may be calculated to be about 0·000013A.

**53. The Electromagnetic Theory.** Having surveyed the whole gamut of the spectrum which has been revealed to scientific researches, we next proceed to find out a unifying theory which would embrace within its scope the complete field of radiation. "To fill all space with a new medium whenever a new phenomenon is to be explained is by no means philosophical but if the study of two different branches of science has independently suggested the idea of a medium, and if the properties which must be attributed to the medium in order to account for the electromagnetic phenomena are of the same kind as those which we attribute to the luminiferrous medium in order to account for the phenomena of light, the evidence for the physical existence of the medium will be considerably strengthened".* Thus to account for the propagation of light through space with a velocity of $3\times10^{10}$ cms. per second, the space was supposed to be filled with an elastic solid (the luminiferous ether), so that the light vibrations are supposed to be of mechanical nature. From mathematical considerations it was established by Maxwell that the velocity of propagation of an electric and magnetic disturbance through space which constitutes the electro-magnetic waves is also exactly equal to $3\times10^{10}$ cm per second. This velocity of electro-

*Clerk Maxwell, Electricity and Magnetism. Vol II P. 431

magnetic waves has been measured in the laboratory indirectly by comparing the magnitudes of capacity of the same condenser measured in electrostatic and electromagnetic units. This necessarily requires the space to be an electrical as well as a magnetic field which possess respectively potential and kinetic energies. Now, potential energy of an electro-magnetic field is due to electric charges at rest and the kinetic energy of the same field is due to the same charges in motion. The logical conclusion which suggests itself is that the space (or the so-called void) is filled with electricity and that material media are only natural boundaries of the space. This, of course, presupposes that space is the ultimate infinite receptacle of energy and that energy cannot exist as such without association with something.

We have seen that the electro-magnetic waves are similar to light waves in respect of their velocity of propagation through space. We now proceed to prove that the electro-magnetic waves can, like light waves, be reflected, refracted, dispersed, and can also exhibit the phenomena of interference, diffraction and polarisation. In the following paragraphs we shall describe Fleming's method of generation and detection of electro-magnetic waves.

**The generator.** It consists of a flat zinc box T (Fig. IX-1) with an open end provided with hollow trunnions supported on vertical uprights. Inside the hollow trunnions there are two

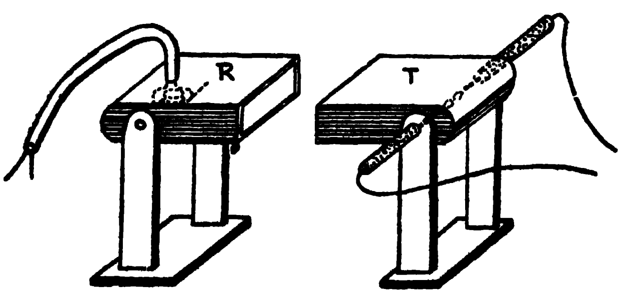



Fig. IX-1

ebonite tubes, through which pass two brass rods each about four inches in length and terminating in brass ball one inch in

diameter. Each of the brass rods is connected to a closely wound spiral covered with gutta-percha called the choke coil. The balls are placed inside the zinc box 1/16 inch apart. The two brass rods are connected to the terminals of the secondary of an induction coil. When the induction coil is set in action, a series of sparks passes between the balls and electrical oscillations are set up in the secondary circuit producing electro-magnetic waves radiated into space.

**The detector.** It is made up of a coherer consisting of two short nickel wires passing through a hollow disc of ebonite without touching (Fig. IX-2). The portions of the nickel wires inside the hollow of the ebonite are just covered with a small

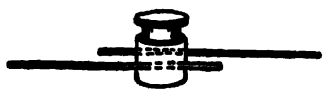


Fig. IX-2

quantity of fine nickel filings. The coherer is fitted on a board inside a zinc box R (Fig. 1) similar to that used with the generator. To the top of the zinc box is soldered a lead pipe without joint through which pass two insulated wires each of

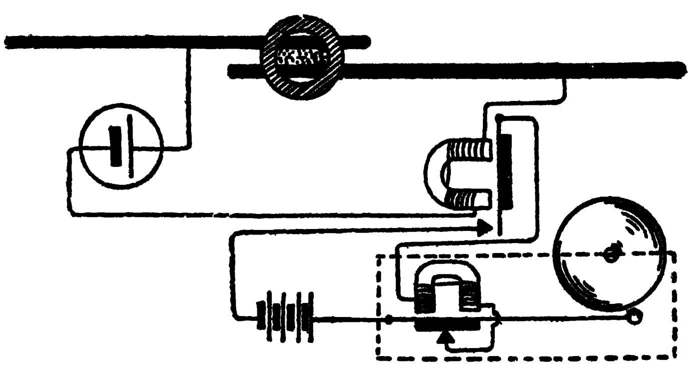


Fig. IX-3

which is connected to one end of the nickel wires. The other end of the lead-pipe is soldered to a metal box containing cells, a relay and an electric bell. The electric connections inside the

metal box is shown in Fig. IX-3. The two connecting wires inside the lead pipe are joined to two choke-coils of good many turns at the points where they enter the metal box.

The coherer is connected in series with a dry cell, the choke-coils, and the relay which closes the circuit of the battery of cells connected to an electric bell. The fine nickel filings spreading between the nickel wires are bad conductors of electricity. They however become conducting when the electro-magnetic radiations from the generator fall on the filings. The radiations falling on the nickel filings cause them to stick or cohere together so that a current can flow between the nickel wires and thus cause the electric bell to ring. If now a smart rap is given to the detector box, the orderly arrangement of the nickel filings is disturbed and the current immediately stops. We now proceed to describe some experiments to establish the similarity between light waves and electro-magnetic radiations.

**Rectilinear propagation.** Place the radiator box and the detector box with their open ends facing one another a few feet apart as shown in Fig. IX-1. As soon as a spark passes between the knobs in the generator-box, the electric bell in the detector-box begins to ring. If the detector box is tilted slightly about the trunnions, so that the two boxes no longer face one another, the electric bell will not ring. This proves that the radiations from the generator-box move in straight lines.

**Transparency and opacity.** Again place the generator and detector boxes facing one another as in the experiment described above and set the generator in action. If we hold a sheet of metal such as a plate of iron, tin-foil, copper-foil, silver-leaf, the hand etc. in the space between the generator and the detector, the bell will not ring. Many liquids such as, salt water, fresh water, solution of soda, methylated spirit can be shown to be opaque to electro-magnetic radiations. But substances like sheets of paper, wooden board, a sheet of glass, slabs of wax, bitumen, sulphur, marble slab, paraffin oil, olive oil and turpentine are all transparent to electro-magnetic radiations. It will thus be

seen that some substances which are quite opaque to light rays are perfectly transparent to electro-magnetic radiations.

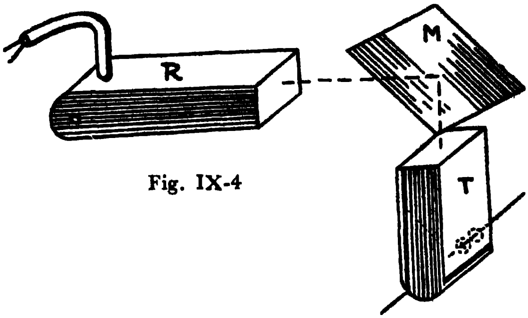



Fig. IX-4

We may thus draw a general conclusion—*all substances which are good conductors of electricity are opaque to electro-magnetic radiations; on the other hand, non-conductors of electricity are transparent to such radiations.*

**Reflection of electro-magnetic waves.** For reflection of electro-magnetic waves any metal surface may be used as reflector. Place the generator-box and the detector-box in positions shown in Fig. IX-4 with their mouths in the vertical and horizontal positions. Setting the generator in action, the bell of the detector would ring only when a metal plate is placed at an angle of 45° in the path of electro-magnetic waves.

**Refraction of electro-magnetic waves.** We now place a prism of paraffin wax in the space between the generator and the

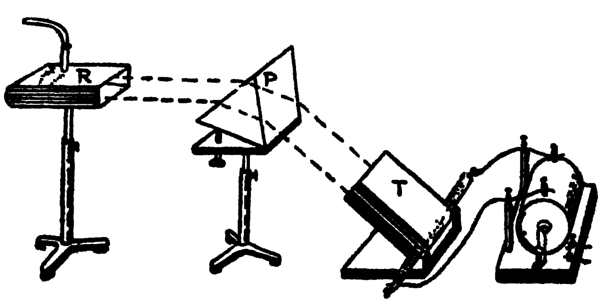



Fig. IX-5

detector which are set at an angle to one another in the manner shown in Fig. IX-5. It will be found that only for a parti-

cular orientation of the prism relative to the generator and the detector, the bell will ring. Hence the electro-magnetic rays are deviated in the same way as light rays on being refracted through the prism. For the particular wave-length used by

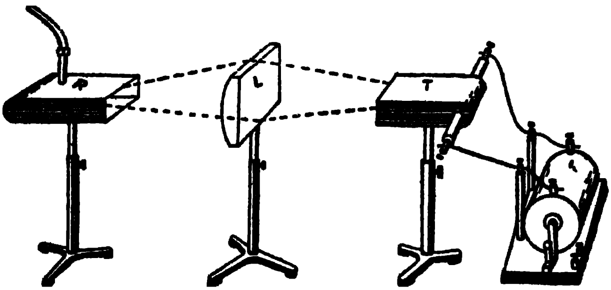


Fig. IX-6

Fleming the refractive index of paraffin was found to be 1·64. Freezing water inside a vessel of the shape of a prism and on experimenting with the dry prism of ice, Fleming found that the refractive index of ice was equal to 1·83.

By placing a paraffin lens cut into the shape of a semi-cylinder in the path of the electro-magnetic radiations, Fleming proved that the rays could be brought to a focus in the same way that light rays can be focussed by a glass-lens (Fig. IX-6).

**Polarisation of electro-magnetic waves.** Make a wire grating by stretching some wires across a wooden frame in the manner shown in Fig. IX-7, the space between the wires being about one-fourth of an inch. Place the grating in the space between the generator and the detector. It will be found that the grating is perfectly transparent to the radiations falling on it when the wires of the grating are perpendicular to the length of the brass-rods carrying the brass-balls. If the grating is rotated into the position where the grating wires are parallel to the brass-rods, then the grating will be perfectly opaque to the radia-

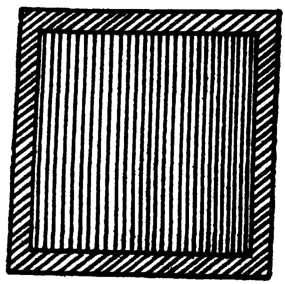


Fig. IX-7

tions. This experiment shows that the radiations emitted by the generator are plane-polarised, the plane of vibration being perpendicular to the length of the spark-gap between the brass-balls inside the generator-box.

Since all the different types of radiations, electro-magnetic, heat, light, ultra-violet rays, X-rays, $\gamma$-rays etc, have one feature in common, namely they are of the nature of waves, and since their common vehicle is the ether-substance, we are forced to the conclusion that all these different radiations are of the same kind, that is, they are electro-magnetic waves. Here, of course, we have to extend our idea of waves—which really mean periodic waxing and waning of some state in space and time. The different types of radiations differ from one another in respect of their wave-length and frequency only.

**54. Kinds of Spectra.** The spectra that are generally observed can be broadly divided into two classes:

(1) **emission spectra** which is produced by energy released by a substance in its state of excitation under the influence of high temperature or high electric pressure.

(2) **absorption spectra.** This is produced by absorption of a portion of the energy falling on the medium under consideration from outside.

55. **The Emission Spectra.** The emission spectra are divided into three classes:

(*a*) **The continuous spectrum.** In this type of spectrum the frequency (or colour in the visible spectrum) changes by imperceptible gradations from one value to the next without any gap or discontinuity. Such spectra are characteristic of incandescent solids or liquids such as, glowing filament lamp,

the craters of the arc lamp, molten metals of high melting point and also of incandescent gases at high pressure which exist in lower levels in stars.

(*b*) **Bright line spectrum.** This consists of a number of lines of different intensities and different frequencies (or colours in the visible region) with intervening dark spaces or spaces of zero intensity. These lines are produced by incandescent gases at low pressure as in a partially evacuated glass tube, in flames containing vapours (sodium flame, for example), in so-called gaseous nebulae (as the irregular nebula in the Orion), and in the tails of comets. Line spectra are also produced by gases at low pressure inside a glass tube, through which heavy electric discharge is passed (the discharge tubes or Geissler's tubes). The spectrum of only the edge of the sun's disc seen through a spectroscope is line spectrum. Such spectrum is characteristic of atomic state of matter. Hence line spectra are also called atomic spectra.

(*c*) **The band spectrum.** It consists of a number of broad bands, sharp at one edge and gradually diffuse at the other end. The sharp end is called the head of the band which may lie on the violetside or onthe red side, that is, towards the shorter or the longer wave-length side. The diffuse end is called the tail ofthe band. Band with a head on the violet side is also said to be degraded towards the red and that with head on the red-side is said to be degraded towards the violet. Examined under high dispersion each band appears to consist of a group of fine lines which are more numerous and closely packed towards the head of the band. Such spectrum is characteristic of molecules. For heavy molecules, such as, $MnO_2$, $S_4$ etc. bands are more closely spaced than for light molecules such as $CO$, $N_2$ etc. Band spectra of $H_2$ are so widely spaced that they overlap and appear like atomic spectra. Band spectra can be obtained only under carefully controlled exciting conditions, otherwise, the molecules would break up into the atomic constituents and atomic spectra would result.

**Characteristics of line spectra.** The lines in the spectrum of an element are found to differ materially in appearance particularly in the case of alkali metals. In general, four different groups of lines may be easily recognised. These are called principal, sharp, diffuse and fundamental represented by letters P. S. D and F (Fig. IX-8.) This designation originally arose from the fact that some of the spectral lines are very

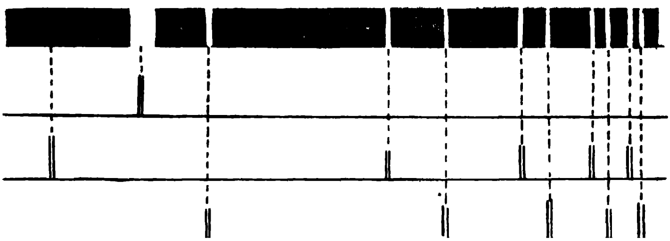


Fig. IX-8

intense (principal), some are very narrow (sharp) while others are quite broad (diffuse). The term fundamental arose from the mistaken notion that the frequencies of these lines are the lowest possible. These terms are still retained in spectroscopic nomenclature but they are used in a different sense to indicate a group of lines whose characteristics may be radically different from those originally signified by these terms. They are now used to indicate the various possible states of excitation of the atom.

**The spectral series.** The wave-lengths of the lines belonging to a particular group can be expressed as a mathematical series. In 1885 J. J. Balmer, a Swiss physicist showed that the wave-lengths of some of the hydrogen lines could be fitted into the formula

$$\lambda = 3645. \frac{n^2}{n^2-4} \qquad \ldots \quad (55.1)$$

where $n$ is an integer equal to or greater than 3. The following table shows how closely the observed values of the wave-lengths of the hydrogen lines agree with those calculated from the above formula. In the above formula $\lambda$ is expressed in Angstrom unit.

| Nomenclature | $n$ | $\lambda$(air) (obs.) | Mean values(obs) | $\lambda$(air) (cal.) |
|---|---|---|---|---|
| H$\alpha$ | 3 | 6562·8473<br>6562·7110 | 6562·7796 | 6562·793 |
| H$\beta$ | 4 | 4861·3578<br>4861·2800 | 4861·3189 | 4861·327 |
| H$\gamma$ | 5 | 4340·497<br>4340·429 | 4340·463 | 4340·466 |
| H$\delta$ | 6 | 4101·7346 | 4101·738 | 4101·738 |
| H$\epsilon$ | 7 | 3970·0740 | 3970·0740 | 3970·075 |
| H$\zeta$ | 8 | 3889·0575 | 3889·0575 | 3889·052 |
| H$\eta$ | 9 | 3835·397 | 3835·397 | 3835·387 |
| H$\theta$ | 10 | 3797·910 | 3797·910 | 3797·900 |

**The Rydberg number.** To fit in with the theoretical deduction of the formula for spectral series, it is necessary to express the Balmer's formula as a reciprocal of $\lambda$ in cm. We thus get

$$\frac{1}{\lambda} = \frac{1}{3645 \times 10^{-8}}\left[1 - \frac{4}{n^2}\right]$$

$$= \frac{4 \times 10^8}{3645}\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

or $$\nu = 109{,}700\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \qquad \ldots \quad (55.2)$$

where $\nu = 1/\lambda =$ the wave-number or number of wave-lengths per centimetre. This numerical 109,700 is represented by $R$ and is called the Rydberg number, since the form of the Balmer's

formula is due to a Swedish scientist J. R. Rydberg. The general formula obtained from theoretical considerations is

$$\nu = Z^2 R \left[\frac{1}{m^2} - \frac{1}{n^2}\right] \quad \dots \quad (55.3)$$

where $Z$ is the atomic number of the radiating atom and $m$, $n$ are integers. Applied to hydrogen atom ($Z=1$) the general formula becomes

$$\nu = R \left[\frac{1}{m^2} - \frac{1}{n^2}\right] \quad \dots \quad (55.4)$$

If we put $m=1$ and $n=2, 3, 4, 5$, etc., so that

$$\nu = R \left[\frac{1}{1^2} - \frac{1}{n^2}\right] \quad n=2, 3, 4, 5 \text{ etc.}$$

we get the series of spectral lines first observed by Professor Theodore Lyman, an American physicist. Hence it is called Lyman series. Similarly,

Balmer series (for Hydrogen) $\nu = R \left[\frac{1}{2^2} - \frac{1}{n^2}\right]$ $n = 3,4,5,6$ etc.

Paschen series ,, ,, $\nu = R \left[\frac{1}{2^2} - \frac{1}{n^2}\right]$ $n = 4,5,6,7$, etc.

Brackett series ,, ,, $\nu = R \left[\frac{1}{4^2} - \frac{1}{n^2}\right]$ $n = 5,6,7,8$ etc.

Pfund series ,, ,, $\nu = R \left[\frac{1}{5^2} - \frac{1}{n^2}\right]$ $n = 6,7,8,9$ etc.

The last three series lie in the infra-red region of spectrum. The above gives the simplest possible example of the series relation. Most spectra are generally very complex containing thousands of lines; nevertheless, they can all be fitted up in a series formula. But a great deal of skill is required for manipulation of the spectroscopic data.

**Fine structure of spectral lines.** When examined with spectroscopes of high resolving power most of the spectral lines are found to consist of a number of lines. Thus each of the first few members of the Balmer series resolve into two lines or doublets. Theory predicts that all atoms with a single valence

electron would produce spectra having a multiplet structure. Some of these have been observed under special circumstances. A single spectral line also breaks up into a number of lines under the influence of strong magnetic field (the Zeeman effect) and electric field (the Stark effect), the components thus produced being polarised.

**Emission spectra and intensity of excitation.** The nature of the spectrum as well as the intensity-distribution emitted by a substance depends on the strength of excitation. Thus, to take an example, at the ordinary temperature of the flame of a bunsen burner; a small quantity of common salt introduced in the flame causes the flame to emit the familiar sodium lines. This does not mean that other lines are not emitted. But the other lines are so faint that they are completely lost to view. If a small quantity of common salt is introduced in the crater of a carbon arc whose temperature is very high, then these faint lines become very prominent and at the same time the spectrum becomes very complex. This is in accordance with Wien's displacement law

$$\lambda_{max} T = \text{constant for all substances}$$

where $\lambda_{max}$ is the wave-length of radiation with maximum intensity in the spectrum of a body at absolute temperature T. Hence with rise of temperature the region of maximum intensity shifts towards the shorter wave-length side.

**Bohr's theory of line spectra.** The electro-magnetic theory of light which brings the whole region of spectrum under its domain, however, fails completely to explain the formation of line spectrum. According to the electro-magnetic theory an oscillating electric charge would dissipate its energy into ether in the same way that a freely oscillating pendulum dissipates its energy into the surrounding atmosphere and is finally brought to rest. Applied to the atomic model of Ernest Rutherford according to whom an atom consists of a central positively charged nucleus with electrons revolving round it, this would mean that in the process of radiation, the orbit of the electron would continually shrink till the electron would fall into the nucleus and

thus be annihilated. This, of course, explains the production of continuous spectrum, but at the same time makes the existence of matter impossible. This difficulty combined with other difficulties such as (*a*) explaining the distribution of energy in the spectrum of a black body at a given temperature, (*b*) specific heat of bodies at low temperatures, (*c*) explanation of photo-electric phenomena led Professor Max Planck of the University of Berlin in the year 1900 to introduce the idea of quantum of energy. According to Max Planck

(*a*) the exchange of energy between matter and ether can take place only in multiples of certain units ;

(*b*) the unit of energy is proportional to the frequency $\nu$ and is equal to $h\nu$ where $h$ is the constant of proportionality called **Planck's constant.** Its value is $6{\cdot}55\times10^{-27}$ erg.sec. The unit of energy $h\nu$ is called a **Photon.**

Planck himself applied the idea of quantum to modify Rayleigh-Jeans formula of distribution of energy in the spectrum of a black-body and got excellent agreement with experimental results. Professor Einstein applied the quantum idea to explain the variation of specific heat with temperature. Much closer agreement with the experimental results was obtained by Professor Debye. The explanation of photo-electric effect was given by Professor Einstein which was based on the idea of quantum of energy.

Professor Neils Bohr in 1913 applied the idea of quantum to the atomic model of Rutherford to explain the origin of line spectrum.

Bohr's theory of atomic spectra is based on two fundamental postulates. These are :

(1) **Condition of stationary orbits.** According to the electro-magnetic theory, the orbital radius of the electron changes continuously in the process of radiation, so that, the orbit of the electron is a flat spiral. As we have seen, this makes the atom unstable. To aviod this difficulty Bohr assumes that the orbital radius of the electron can have only

certain quantised values. This means that the electron can rotate in certain favoured orbits and while rotating in any of these orbits, the electron does not radiate energy. Hence these quantised orbits are said to be stationary.

(2) **Bohr's frequency condition.** The atom with the electron rotating in a quantised orbit possesses a certain definite amount of energy. Only when the electron jumps from the orbit of higher to the lower energy state of the atom, the difference of energy in the two states, is given to the ether as a quantum of energy. If $E_2$ is the energy in the higher stationary state and $E_1$ is the energy in the lower stationary state, the frequency $\nu$ of the energy emitted by the atom is

$$E_2 - E_1 = h\nu$$

so that $$\nu = \frac{1}{h} \cdot (E_2 - E_1)$$

In terms of wave-number $$\bar{\nu} = \frac{\nu}{C} = \frac{E_2}{hC} - \frac{E_1}{hC}.$$

where $C$ is the velocity of light.

Comparing this result with Eqn. (55.4) we find that the energy of the hydrogen atom in the $N$th excited state is

$$E_N = \frac{hCR}{N^2},$$

where $R$ is the Rydberg number* and $N$ is an integer. Substituting the values of $h$, $C$ and $R$,

$$E_N = \frac{6 \cdot 55 \times 10^{-27} \times 3 \times 10^{10} \times 109678}{N^2}$$

$$= \frac{2155172 \cdot 7 \times 10^{-17}}{N^2} = \frac{21 \cdot 55 \times 10^{-12}}{N^2} \text{ (approximately)}$$

Since in the normal or equilibrium state the energy of an atom must be the minimum possible, the expression for energy of the atom in $N$th state is

$$E_N = -\frac{21 \cdot 55 \times 10^{-12}}{N^2} \text{ ergs} = -\frac{21 \cdot 55 \times 10^{-12}}{N^2 \times 1 \cdot 59 \times 10^{-12}} \text{ electron-volt}$$

$$= -13 \cdot 55 \text{ electron-volt,**}$$

*The accepted value of Rydberg number for hydrogen is (109,677·750 $\pm$0·05) per centimetre.

**One electron-volt is the amount of energy required to move an electron through a potential difference of 1 volt.

$$\therefore 1 \text{ electron volt} = \frac{4 \cdot 77 \times 10^{-10}}{300} = 1 \cdot 59 \times 10^{-12}$$

**The energy diagram.** Giving different values to $N$ we can draw a number of possible energy levels of the atom. By absorption of energy the atom is raised from the normal ($N=1$) to a higher energy level. From this higher energy level the atom can fall to any lower energy level radiating the excess energy which appears as a spectral line. These energy levels are shown in Fig. IX-9.

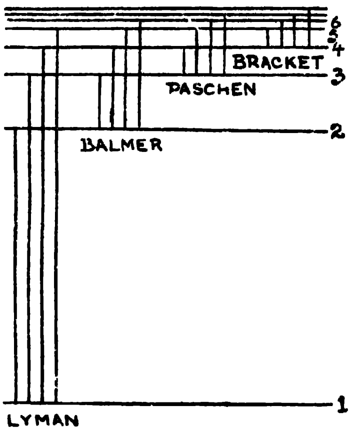



Fig. IX-9

**The quantised orbits.** It can be proved that the radius of the Nth stationary orbit for hydrogen atom is given by

$$r_N = \frac{N^2 e^2}{2hC} \times \frac{1}{R}$$

where $e$ is the electronic charge and $N$ is an integer. In the normal energy state of the atom, the electron rotates in the orbit corresponding to the value of $N=1$. This is called the K-orbit. The orbits corresponding to $N=2,3,4,5,6$, are called respectively the L,M,N,O,P orbits. When the atom is excited by some external cause, the electron is raised from the K-orbit to some other higher orbit from which it can drop to any inner orbit. The emission of hydrogen lines due to possible transitions is shown in Figs. IX-9 and IX-10.

**Line spectrum and band spectrum.** While line spectrum of the atom is due to jump of the electron between possible orbits, the band spectrum is produced by changes in (1) rotational

energy of the molecule as a whole, (2) vibration of the atoms constituting the molecule and (3) electronic transition of the individual atoms of the molecule. Hence the

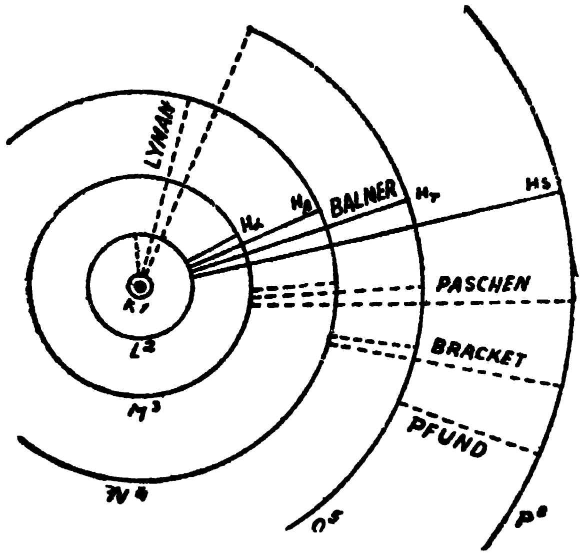



Fig. IX:10

whole band spectrum may be resolved into (1) rotational spectrum (2) vibrational spectrum and (3) electronic spectrum. These energies of rotation, vibration, and electronic transition can also be quantised.

**The width of spectral line.** (*a*) Theoretically a spectral line should be a geometrical line. This would be true if only one atom were responsible for the production of one spectral line. But a source of light consists a of multitude of atomic sources of which a certain number are jumping from one energy level to another at a given instant. The intensity of a spectral line will necessarily depend on the number of atoms taking part in the given transition. Further, the energy levels may have a thickness, however small. This will have the effect of broadening the spectral lines.

**(*b*) Broadening due to Doppler effect.** The radiating atoms of the source are not stationary. Some of them are moving

towards and some are moving away from the spectrometer with velocities given by Maxwell's distribution law. It will be proved later on that when a radiating source moves towards an observer, the frequency of the radiation is increased, the amount of increase depending on the relative velocity between the source and the receiver. Conversely, if the source moves away from the receiver, the frequency is reduced. This is known as Doppler's effect. Since change of frequency means shifting of the line along the spectrum, the spectral line will be broadened, the amount of broadening depending on the range of velocities of the radiating atoms.

(*c*) **Broadening due to collision.** While radiating, the atoms may collide with one another which may affect the amplitude and phase of the radiations emitted by the colliding atoms.

(*d*) **Broadening and asymmetry by pressure.** Experiment shows that the increase of pressure of a gas radiating energy has the effect of widening the spectral line and also producing some amount of asymmetric distribution of frequency on the two sides of the point of maximum intensity. This might probably be due to the interaction of the electric fields due to radiating atoms.

It was discovered by Humphreys and Mohler in 1895 that when a source of light is subjected to pressure the spectrum shifts towards the red end, the amount of shift being directly proportional to pressure. Hence, for comparison, the wave-lengths determined in the laboratory should be corrected for pressure variation.

**Origin of continuous spectrum.** The idea of broadening of the spectral lines due to pressure of the gas leads to a probable explanation of the origin of continuous spectrum. In the case of a liquid and a solid the radiating atoms are very close together and their interaction is very great. Accordingly the lines may be broad enough to overlap, thus giving the appearance of a continuous spectrum.

**56. The Absorption Spectrum. Classification of absorption Spectra.** When a strong beam of white light is sent through a material medium and is then examined by a spectroscope, the spectrum thus produced is generally found to be crossed by a number of dark bands or lines. This is due to some of the frequencies being absorbed by the material medium. It has been found that every substance has its characteristic region of absorption which may lie in the infra-red, visible, or ultra-violet region of the spectrum. If the absorbing substance is a solid or a liquid, these regions of absorption are broad bands without any structure. With gases as absorbing media the regions of absorption are usually sharp lines. Gases may also produce band absorption spectra, because when examined by a spectroscope of high resolving power, these dark regions break up into a large number of dark lines separated by narrow illuminated spaces arranged in an orderly manner. In fact band spectrum can be best studied with the help of absorption bands, since in the process of production of emission bands, the molecules break up into constituent atoms unless the condition of production is very carefully controlled. Thus, like emission spectra, the absorption spectra can also be classified into three groups (*a*) continuous absorption, (*b*) line absorption and (*c*) band absorption spectra.

**The solar spectrum.** In 1802 Wollaston observed that the spectrum of the sun-light was crossed by a certain number of dark lines but he could offer no explanation. In 1814 Joseph Von Fraunhofer an optician in a firm in Munich observed as many as 576 dark lines in the solar spectrum. He assigned letters A,B,C,D,E,F,G,$H_\gamma$ and $H_\delta$ to some of the prominent lines. These lines are situated in particular colour zones in the spectrum and to specify a particular colour in an emission spectrum, the dark line in the solar spectrum situated in the region of colour under consideration is generally referred to. By illuminating half of the spectrometer slit with sodium light and the other half by sun-light (so that the two spectra were placed one above the other) he could identify the D-line of the solar spectrum with the yellow sodium emission line.

It was soon discovered that the dark lines C, F, $H_\gamma$ and $H_\delta$ were identical with the emission lines of Hydrogen; the dark lines H and K were continuous with two lines in the emission spectra of calcium. The work was continued till all the 576 lines were identified with the emission lines of sodium, hydrogen, barium, magnesium, manganese, calcium, iron, chromium, nickel, cobalt, zinc, copper and aluminium.

The explanation of the appearance of the dark lines in the solar spectrum (now known as the Fraunhofer lines) was given by Kirchhoff in the year 1859. His explanation was based on the following experiment : He placed a bunsen flame fed with common salt in the path of sunlight which illuminated the slit of spectrometer and found that the D-lines of the solar spectrum appeared to be darker and broader when the sun-light passed through the sodium flame than in its absence. He then replaced the sun-light by a source of white light and again observed the dark line in the same position as previously occupied by the D-line of the solar spectrum. On removing the source of white light, a yellow line appeared in the position previously occupied the dark line. The natural conclusion was that the same sodium vapour which emits the sodium light can also absorb the same colour when light containing that colour is made to pass through its vapour. For the success of the experiment it is necessary that the temperature of the sodium flame must be lower than that of the source of white light, because the sodium flame also emits the yellow line due to its high temperature. The dark line thus observed is not absolutely dark, but it appears dark in contrast with much brighter back ground. The most convincing proof of absorption of sodium line by sodium vapour was given by Roscoe. He heated sodium in a sealed glass tube containing some hydrogen till it vaporised. On looking at a sodium flame through the tube, the tube appeared dark, but on looking at a source of white light through the tube it appeared to be perfectly transparent.

This relation between emission and absorption is not characteristic of sodium vapour alone: the same experiments can be performed with any other substance in the vaporous

condition particularly with alkali halides such as lithium chloride, rubidium chloride etc.

All these experiments described above thus point to the conclusion that a substance *can absorb the same light which it emits.*

**Kirchhoff's law.** This law of emission and absorption of light agrees with a more general thermodynamical law known as Kirchhoff's law which runs as follows.

*At a given temperature the ratio of emissive to absorptive powers is the same for all substances and is equal to the emissive power of a perfectly black body at the same temperature.*

It follows that if the sodium burner at the temperature of the flame emits the yellow line, it must also absorb the same yellow line falling on it at the same temperature. Its power of emission and absorption will, of course, depend on the temperature of the sodium vapour. Let $J$ be the intensity of light falling on the sodium vapour $J'$, the intensity of light emitted by the sodium vapour and $a$, the absorptive power of the sodium vapour. Then, the intensity in the region of the D-line of the spectrum would be $(J-aJ+J')$ when the sodium vapour is placed between the source of white light and slit of the spectrometer. The quantity within bracket would be greater or less than $J$ according as $(J'-aJ)$ is positive or negative. The intensity of the D-line would be less than $J$, if $J'$ is less than $aJ$. Since the emissive power of a source depends on temperature, this requires that the temperature of the source of white light must be sufficiently higher than that of the sodium vapour.

We are now in a position to explain the occurrence of Fraunhofer lines in the solar spectrum. The sun is supposed to consist of a central hot body (the nucleus) which is surrounded by hot vapours 500 to 1000 miles thick. This enveloping layer is called the *photosphere.* Surrounding the photosphere there is another glowing mass of vapour which is 5000 miles to 10000 miles thick. This layer is called the *chromosphere,* the corona, or the reversing layer. The chromosphere contains vapours of substances existing in the photosphere. The temperature of the

chromosphere is much less than that of the photosphere. When light emitted by the photosphere passes through the cooler chromosphere the latter absorbs the light characteristic of the substances existing in the photosphere and the absorbed light appears as dark lines in the solar spectrum.

At the time of total solar eclipse the photosphere is completely cut off, so that only light reaching the earth is from the chromosphere. When this light is examined by the spectroscope, bright lines characteristic of the elements present in the chromosphere are observed in the positions occupied by the Fraunhofer lines.

**57. The Doppler's principle.** It is found that when a source of sound approaches an observer, the pitch of the sound appears to rise and when the source recedes from the observer, the pitch appears to fall. The change in pitch due to this cause can be calculated as follows. For the sake of generality, we shall suppose in what follows that the source, the recipient, and the medium carrying the waves are all in motion and that they are all moving in the same direction. All velocities

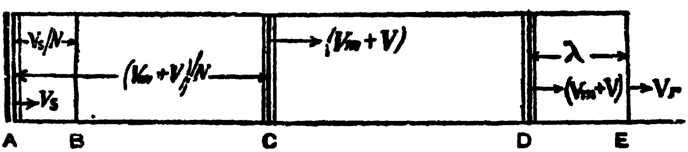



Fig. IX-11

will be measured relatively to some fixed embankment which is here the surface of the earth. Let $V_s$, $V_r$, $V_m$, be the velocities of the source, recipient, the medium respectively and let $V$ be the velocity of the waves with respect to the medium. Let $N$ be the frequency of the sound emitted by the source when stationary and $\lambda$, its corresponding wave-length. In Fig. IX-11 A is the position of the source at the instant when it emits a particular wave-front. When it emits the next wave-front, it will have moved to the position B, where the distance AB is equal to $V_s/N$. But during the interval $1/N$, the wave-front

emitted at A has travelled to the point C, where the distance AC is equal to $(V_m+V)/N$. Hence the altered wave-length is

$$\lambda' = AC - AB = \frac{V_m+V-V_s}{N}$$

The waves of length $\lambda'$ continues to move with the velocity $(V_m+V)$ relative to the earth.

Coming to the recipient, let E be its position when it receives a particular wave-front, the next wave-front will at this instant be at D which is at a distance $\lambda'$ behind E. Since the relative velocity of the waves with respect to the recipient is $(V_m+V-V_r)$, it will receive the next wave after an interval of time equal to $\lambda'/(V_m+V-V_r)$. Hence the number of waves falling on the recipient per second is

$$N' = \frac{V_m+V-V_r}{\lambda'} = N\frac{V_m+V-V_r}{V_m+V-V_s} \quad \dots \quad (57{\cdot}1)$$

where $N'$ is the altered frequency.

**The Fizeau-Doppler's principle.** It was Fizeau who first successfully applied the Doppler's principle to optical cases. In the case of light waves the medium is the ether which is permanently at rest, so that $V_m$ in equation (57.1) is zero. Hence

$$N' = N\frac{V-V_r}{V-V_s} \quad \dots \quad (57{\cdot}2)$$

If, further, the recipient is stationary and the source moves away from the recipient, we have $V_r = -0$ and

$$N' = N\frac{V}{V+V_s} \quad \dots \quad (57{\cdot}3)$$

Under the same condition, if the source moves towards the recipient,

$$N' = N\frac{V}{V-V_s} \quad \dots \quad (57{\cdot}4)$$

In terms of wave-lengths, Eqn. (57.3) can be written as

$$N' = \frac{V}{\lambda'} = \frac{V}{\lambda}\cdot\frac{V}{V+V_s}$$

or

$$\lambda' = \lambda\frac{V+V_s}{V} = \lambda\left(1+\frac{V_s}{V}\right) \quad \dots \quad (57.5)$$

Similarly Eqn. (57.4) can be written as

$$\lambda' = \lambda\left(1-\frac{V_s}{V}\right) \quad \dots \quad (57{\cdot}6)$$

From Eqns. (57.5) and (57.6)

$$d\lambda = \lambda' - \lambda = \pm\frac{\lambda V_s}{V} \quad \dots \quad (57{\cdot}7)$$

Hence knowing $\lambda$, $d\lambda$ and $V$, the velocity of the source relative to earth may be calculated. The positive sign indicates that the source is moving away from the observer, while the negative sign indicates that the source is moving towards the observer.

Applied to optical cases Fizeau-Doppler's shift would mean that a spectral line emitted by a source moving towards the spectrometer would be shifted towards the violet end, its colour at the same time changing. If the source moves away from the spectrometer, the shifting would be towards the red end. Belopolsky verified this fact experimentally in the laboratory by reflecting light from a source with the help of moving mirrors mounted on the rim of a wheel rotating at high speed. The experiment was repeated in 1907 by Prince Galitzin and J. Wilip who replaced the spectrometer by Echelon spectroscope which enormously magnified the amount of shifting.

Another interesting method of verification of the principle was that adopted by Stark in determining the velocity of canal rays. These are positively charged particles passing through a perforated cathode and are, therefore, deflected by electric and magnetic fields. The velocities of these particles can be determined from their deflection in the electric and magnetic fields. The light emitted by these particles was examined by a spectroscope and from the Fizeau-Doppler shift their velocities were calculated which agreed with the value calculated from deflection in electric and magnetic fields.

**Applications of Fizeau-Doppler's principle to astronomy. The rotation of the sun.** With the help of this principle very convincing conclusions were drawn regarding the state of motion of the outermost layer of the solar atmosphere. By observing the spectrum of the sun-spots, the velocity of the atmosphere at different latitudes of the sun can be calculated. This is done by comparing the position of a spectral line when the sun-spot is moving perpendicularly to the line of vision near the centre of the sun's disc (which produces no shift) with the position of the same line when the sun-spot is near the edge of the disc which

corresponds to the motion of the sun-spot in the line of vision. By observation of spectra of sun-spots at different latitudes it was proved that the equatorial zone of the sun rotates uniformly with a period of 27 25 days while the period of rotation of the zones at different heights from the equator increases with distance from the equator It follows that the sun does not rotate as a solid body

**The spectroscopic binaries.** These are pairs of stars which are so close together that they appear as single stars even with the telescope of highest resolving power Such stars are divided into two classes—(1) a pair consisting of a dark and a bright star and (2) a pair consisting of two bright stars Each pair rotates about its common centre of gravity Observed by a spectrometer the binaries of class (1) would give a single line which moves periodically about a fixed point The binaries of class (2) give lines which break up into two and again coalesce into a single line periodically In this way many hundreds of spectroscopic binaries have been observed

**Nature of saturn's rings.** When viewed through a telescope the planet saturn is found to consist of a heavy central mass which is encircled by three flat concentric rings The innermost ring called the crape ring is only faintly visible The middle and the outermost rings are separated from one another by a dark space called the Cassini's division There was much controversy regarding the composition of saturn's rings One school of physicists held that the rings were solid, while the other school maintained that it consisted of swarm of particles closely packed and held together by the gravitational attraction of the central mass of the planet

Prof Maxwell proved mathematically in 1857 that a thin solid ring rotating round the central mass of the planet would be dynamically unstable and it would break up into pieces, if the condition of equilibrium were slightly disturbed If the ring were made up of swarm of particles, held by gravitational force of the central mass, then the centripetal force required for rotation in circle would be supplied by the gravitational pull of the

central mass. If $M$ be the central mass, and $m$, the mass of a particle of the ring which is at a distance $r$ from the centre, then

$$G\frac{Mm}{r^2}=r\omega^2$$

where $\omega$ is the angular velocity of the mass $m$. This gives

$$\omega \propto \frac{1}{r^{3/2}}.$$

Hence the angular velocity of a mass near the inner edge of the ring would be greater than that near its outer edge. By using Fizeau-Doppler's principle Kepler actually proved in 1895 that the inner edge of the ring moves faster than the outer edge.

**Rotation of planets.** By examining the shift in the spectrum of sun light reflected from the surface of planets particularly venus and mercury it has been concluded that venus and mercury always turn the same face towards the sun and, therefore, they have no rotation about their axes like the earth.

**Temperature of heavenly bodies.** It has already been mentioned that the nature of spectrum as well as distribution of intensity in the spectrum depends on the temperature of excitation (compare Wien's displacement law). Hence from the study of intensity of lines emitted by the self-luminous heavenly bodies, the temperature of these bodies can be estimated. It has been found in this way that the gaseous nebulae which produce very bright line-spectrum are the hottest, their temperature lying between 20,000°C to 30,000°C. Some stars are found to shine with blue-white light, while others are comparatively cooler. Some stars are again found to glow with red light; these are cooler still. These heavenly bodies at different temperatures only mark out the different stages in the eternal cycle of evolution of stars.

**58. Fluorescence.** No material medium is perfectly transparent in the true sense of the term. Accordingly, from the principle of energy, the energy absorbed by a material medium must be transformed.

When light is absorbed by a body a portion of it is always transformed into sensible heat which shows itself as rise of

temperature of the body. Another portion is transformed into radiation of longer wave-lengths. This kind of degradation of wave-length is only a particular case of a more general law first enunciated by Sir George Stokes.

**The Stokes' law.** *When radiation is absorbed and re-emitted by body, the wave-length of radiation re-emitted is generally longer than that of the absorbed radiation.*

**Theory of fluorescent radiation.** A simple theory of fluorescent radiation can be obtained in the following way :—

According to the quantum theory radiation of frequency $\nu$ is equivalent to a swarm of particles called *photons* of mass $h\nu/C^2$, of momentum $h\nu/C$, and of energy equal to $h\nu$ moving with velocity of light $C$. When these photons traverse a medium containing material particles, the photons and the particles of matter collide with each other in the same way that molecules of two gases enclosed in a box collide with each other. There is, however, a fundamental difference between collision of two molecules and collision of a molecule and a photon. The collision between two molecules is always an *elastic collision*; that is, the sum total of kinetic energy and momenta of the colliding particles remains constant before and after collision. The collision between a molecule and a photon may also be an *inelastic collision* in which process not only the kinetic energy and momenta but also the internal energy of the molecule may change; this means that the energy state of the molecule may suffer transition from high to low and from low to high level at the expense of the energy of the colliding photon.

To express this idea mathematically let $\nu$ be the frequency of the incident radiation, so that, the energy of the photon is $h\nu$, let $m$ be the mass of the molecule, and $v$ and $v'$ its velocities before and after collision; let $E_a$ and $E_b$ be the internal energies of the molecule before and after collision; as a result of the collision let the photon change its frequency from $\nu$ to $\nu'$. Then, from the principle of energy

$$h\nu+\frac{mv^2}{2}+E_a=h\nu'+\frac{mv'^2}{2}+E_b. \quad \ldots \quad (58\cdot1)$$

It can be proved from the stand-point of the kinetic theory that in a large number of random collisions between two types of molecules of masses $M_1$ and $M_2$ ($M_1 > M_2$) the kinetic energy gained by the heavier molecule is of the order of $2M_2/M_1$. Since the mass of a molecule is of the order of $10^{-23}$ gm. and the mass of a photon is of the order of $10^{-33}$ for visible light, the fraction of energy of the order of $10^{-10}$ of the energy of the photon is transferred to the molecule. This is a negligible fraction and accordingly $v'$ may be taken to be equal to $v$. Hence

$$\nu - \nu' = \frac{1}{h} (E_b - E_a) \qquad \ldots \quad (58.2)$$

**Cor. 1.** If $E_a = E_b$, the collision is of elastic type and the energy is emitted without change of wave-length.

**Cor. 2.** If $E_b < E_a$, $\nu$ is less than $\nu'$; in this case, the wave-length of re-emitted radiation is less than that of the incident radiation. In this case the colliding photon loses a part of its energy. The corresponding spectral lines are called the *Stokes' lines.*

**Cor. 3.** If $E_b > E_a$, $\nu$ is greater than $\nu'$. In this case the wave-length of re-emitted radiation is greater than that of the incident radiation. In this case the colliding photon gains some energy from the molecule. The corresponding line is called the *anti-Stokes' or Raman lines.*

The fluorescent radiation* can be easily excited in substances like sulphate of quinine, fluor-spar, coal-oil, chlorophyll, eosine, fluorescein, rhodamin, iodine, petroleum-oil. When white light falls on these substances the re-emitted radiations lie in the visible region and can thus be easilly observed. Ultra-violet light falling on the cornea and the teeth causes them to fluoresce strongly. It also causes the nails and the hair to fluoresce though to a smaller extent.

The power of light of exciting fluorescence is reduced after passing through a solution which fluoresces on account of

---

*The name fluorescence is due to the fact that the phenomenon was first observed in fluor-spar which emits blue light when it is irradiated with sun-light.

absorption of incident radiation. Hence the layer of a fluorescent substance first struck by light shows maximum fluorescence. The fluorescent light is not usually of a single wave-length but consists of a band with a pronounced maximum of intensity at a particular wave-length. The wave-length of maximum intensity of radiation in the fluorescent light does not depend on the wave-length of the exciting radiation. It depends only on the nature of the fluorescent substance. The intensity of flourescence of a solution depends upon the character of the solvent.

Fluorescent light in solution is generally polarised, the degree of polarisation depending upon concentration and temperature, being very pronounced in viscous solvents. The degree of polarisation in isotropic substances depends on the obliquity of the direction of emission of the fluorescent light. As already mentioned the fluorescent spectra of solids and liquids are continuous bands. The fluorescent spectra of monatomic gases and vapours consist of a number of lines whose character changes considerably due to the admixture of inert gases.

**59. Scattering of Light.** On watching a log of wood floating on water, it will be found that small ripples striking the log are scattered in all directions while bigger waves move forward unimpeded, causing the log only to move up and down. The light waves through the ether behave in a similar manner with regard to fine particles, such as, dust, frozen moisture air molecules etc. When sun-light falls on these particles waves of smaller lengths are scattered in all directions, while those of longer lengths pass right on. It follows that transmitted light has a colour lying around the red, while blue colour predominates in the scattered beam. That the wave-length of the scattered light depends on the size of the scattering particles can be shown by the following experiment :—

Take a dilute solution of sodium thio-sulphate in a glass basin and place it in the path of a beam of white light. Next add a few drops of dilute sulphuric acid to the solution. This causes fine particles of sulphur to be precipitated which

slowly coalesce and grow in size. On viewing normally to the beam of light, the colour of the light is at first distinctly blue and the transmitted beam shows a yellowish tint; the transmitted beam then becomes orange and finally red. This is due to the fact that as the particles grow in size, waves of larger lengths are added to the scattered beam and there is a deficit of corresponding colours in the transmitted light which, therefore, appears red.

**The glorious sun-set and the blue of the sky.** This phenomemon of scattering is responsible for the colours of the setting sun. When the sun is overhead, the thickness of the atmosphere traversed by the rays is the smallest, but as the sun approaches the horizon, the beam travels through greater and greater thicknesses of the atmosphere. As a result the proportion of colours scattered to those transmitted through the atmosphere increases as the sun sinks down towards the horizon. This produces a change in the colour of the beam transmitted through the atmosphere. While the colour of the setting sun is due to a change in wave-length of the transmitted beam, the blue of the sky is due to the sun-light scattered by molecules of air or other suspended particles. These suspended particles being of larger size, scatter waves of longer length which, superposed on the blue scattered by the air molecules produces a grey colour. This is further borne out by the fact that blue of the sky is deeper at high altitudes where the suspended impurity is much less or after a heavy shower of rain which precipitates all such impurities present in the atmosphere.

**Blue of the sea.**—The blue of the sea can also be explained in a similar manner—that is, by scattering of light by water molecules. In order to prove that this is really the case, some quantity of sea-water W (Fig. IX-12) is taken in a spherical glass bulb. The glass bulb is placed inside a light-tight wooden box which is painted black on the inside. The box is provided with two side holes through which a beam of sun-light may be made to pass.

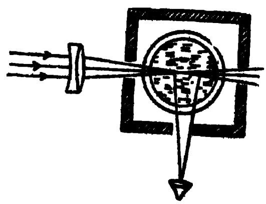


Fig. IX-12

On looking into the flask sideways through another opening on the side, the track of the beam through water is found to be marked out by a beautiful blue colour. The experiment should, of course, be conducted in a dark room.

**Photography with infra-red rays.**—This phenomenon of scattering has very important application in modern photography. Since waves of larger length are scattered less than the smaller waves, the former can penetrate through a larger distance without any appreciable loss in intensity due to scattering. Now objects many miles away appear misty and indistinct even on a clear day, the reason being that visible light coming from these objects are partly scattered and partly absorbed by transmission through this distance. Infra-red rays whose wave-length is much larger than those of the visible light are scarcely scattered at all by the atmosphere. But, then, the eye as well as the ordinary photographic plates are quite insensitive to these rays. Recently photographic plates have been prepared which are sensitive to infra-red rays. Photographs of distant objects taken with the help of these rays are found to be extraordinarily sharp and well defined. Photographs of objects 20 or 30 miles away taken with infra-red rays on these specially prepared plates show details of the object very clearly.

**Rayleigh's equation for Scattering.**—The scattering of light by fine particles was first studied by Tyndall. Lord Rayleigh made systematic investigation of the phenomenon of scattering and obtained a mathematical expression for the intensity of light scattered in a direction making an angle $\beta$ with the direction of incident light at the scattering molecule. The relation is

$$J = A^2 \frac{2n^2(n-1)^2}{N\lambda^4}(1+\cos^2\beta) \qquad \dots \quad (59\cdot1)$$

where $A$ is the intensity of the incident light, $n$ the refractive index of the scattering molecule, $N$ the number of molecules per unit volume and $\lambda$ is the wave-length of the incident light. Thus the intensity of scattered light varies as the inverse fourth power of the incident wave-length. This explains why blue light is scattered much more strongly than red light.

**State of polarisation of scattered light.**—Equation (59.1) holds for unpolarised incident light. Experiment shows that with unpolarised incident light the light scattered in a direction perpendicular to the direction of incident light is almost completely plane-polarised and its plane of vibration is perpendicular to the plane containing the incident and the scattered beams. In any other plane, scattered light is only partially polarised containing a strong component vibrating in a plane perpendicular to the plane containing the direction of incident beam and the direction of observation and a weak component vibrating parallel to this plane. Cabannes showed that if $\rho$ be the ratio of the weak (parallel) to the strong (perpendicular) component, then Rayleigh's formula (Eqn. 59.1) must be multiplied by a factor $(6+6\rho)/(6-7\rho)$

so that $$J=A^2\,\frac{2n^2(n-1)^2}{N\lambda^4}\,\frac{6+6\rho}{6-7\rho}\,(1+\cos^2\beta) \qquad \ldots \quad (59{\cdot}2)$$

in order to obtain expression which agrees completely with experimental results. This equation holds for solids, liquids and gases. In the case of unsymmetrical distribution of electric charges in the scattering molecules, the scattered light is found to be slightly unpolarised in a direction perpendicular to the direction of propagation of incident light.

**60. The Raman Effect.**—It has already been pointed out that in the case of inelastic collision between a photon and a molecule, the photon can gain or lose some amount of energy (Eqn. 58 .2), thus producing what are known as anti-Stokes and Stokes lines. The existence of anti-Stokes line was predicted from the Compton effect (discovered in the year 1923) by Smekal. Compton found that when X-rays were scattered by a solid, the spectrum of scattered radiation showed lines of frequency less than that of the incident frequency in addition to the lines of incident radiation Smekal predicted that not only lines of lower frequency but also those of higher frequency should appear in the spectrum of scattered radiation. In 1924 Kramers deduced a formula of dispersion with the help of the new quantum theory which confirmed Smekal's predictions. These predictions were first verified experimentally by Prof. C.V. Raman, the then Palit Professor of Physics of the University

of Calcutta. The experimental difficulty arose from the low intensity of these lines and to obtain a spectrogram, exposure extending for several days was necessary.

The experimental arrangement as modified by Prof. Wood for photographing Raman lines is shown in Fig. IX-13. This figure gives both the longitudinal and transverse section of the apparatus. It shows a mercury arc B inside a closed box with a small opening. C is a tube containing a solution which isolates a particular line of the mercury spectrum. It also acts as a cylindrical lens which concentrates the light of mercury are on the observation tube A. K is an additional filter which is sometimes necessary to prevent decomposition of the solution in C by the action of ultra-violet rays. The observation tube is backed by an ellipto-cylindrical

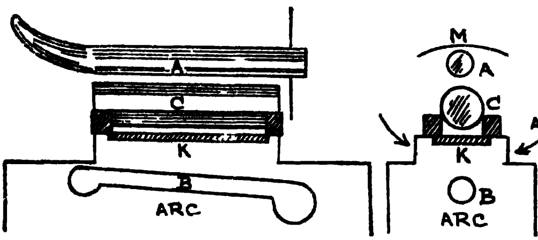



Fig. IX-13

mirror one of whose focus coincides with the arc and the other focus with the observation tube. One end of the observation tube is horn-shaped and is blackened in order that this end of the tube many not reflect any light on to the observation end. The spectograph is placed near the flat end of the observation tube and the axis of the spectrograph is made exactly parallel to the axis of the observation tube. For success of the experiment the slit of the spectrograph must be carefully protected from stray light and the alignment of the axis of observation tube and the axis of the collimator must be perfect. The alignment is secured by observing an unblackened nodule of glass placed on the horn-end of the observation tube and lying exactly on the axis of this tube. This can be illuminated from behind and

observed through the spectrograph. The observation tube may be filled with carefully purified liquid or with powdered solid under examination.

**Characteristics of Raman lines.**—Each line in the exciting radiation is accompanied by a group of Raman lines on the short as well as on the long wave-length side. The lines on the long wave-length side are invariably more intense than those on the short wave-length side. The intensity of the lines on the short wave-length side rapidly decreases as its separation from the exicting line increases. The displaced lines are usually very sharp. In some cases the Raman lines are broader than the exciting lines. Some of the displaced lines are more strongly polarised than that required by the Tyndall scattering while other lines are only slightly polarised. Each group of Raman lines shows the same wave number separation from the chosen parent line. With few exceptions the displaced Raman lines correspond to observed infra-red bands of the substance. But there seems to be little or no correlation between the intensities of the Raman lines and those of the observed infra-red bands. But the Raman lines do not represent all the observed bands in the observed infra-red spectrum of the substance under consideration. The explanation of these anomalies has been given by Langer and Rasetti. The data obtained from the study of the infra-red spectrum and the Raman effect enable us to determine the shape of a molecule.

Difference between fluorescent radiation and Raman radiation lies in the fact that in the case of the former the fluorescent radiation is of the same frequency as that of the radiation which the fluorescing substance can absorb. In the case of Raman radiation the frequency difference $\Delta\nu$ between the displaced radiation and the incident radiation is the same, no matter what the frequency of incident radiation may be.

The difference between ordinary (Tyndall) scattering and modified (Raman) scattering lies in the difference of phase relationship between the undisplaced and displaced lines. In Tyndall scattering there is a definite phase relation between incident and scattered radiations. In the case of Raman

scattering the difference in phase changes from molecule to molecule. It is for this reason that ordinary scattering is coherent while Raman scattering is incoherent.

**61. Phosphorescence.** In another class of substances like sulphide of calcium (known as Balmain's luminous paint) and sulphides of barium and strontium, the bodies continue to emit light for sometime after the exciting light is withdrawn. This class of phenomenon is called phosphorescence.* Some substances like gelatin, horn, egg-shells, paper, etc, phosphorescence feebly at ordinary temperatures, but when cooled to the temperature of liquid air, these substances glow strongly when exposed to white light. In order to produce phosphorescence, some impurity must always be present in these bodies. This suggests that light absorbed by phosphorescent bodies produces some chemical change within the body. During re-emission of light the body slowly returns to its original state of chemical composition. The phenomenon of phosphorescence is only imperfectly understood.

Non-luminescent substances placed in any part of the spectrum assume the colour of that part of the spectrum, but a luminescent body behaves in a different way. Thus, a glass tube containing solution of quinine sulphate with few drops of sulphuric acid appears red when held at the red end of the spectrum and when placed in the green and yellow it appears green and yellow. But when placed in the blue and the violet, it appears of pale blue colour. Similarly, a tube containing solution of chlorophyll continues to emit red light when it is moved up the spectrum. When placed in the violet, the solution emits a brownish light due to admixture of green light as well as red.

Another point of distinction between luminescent and non-luminescent substances is that solution of a transparent non-luminescent substance placed in a glass trough cannot be generally seen when a beam of sunlight is passed through it.

*The term phosphorescence is derived from phosphorus which emits continuous light without any rise of temperature. But phosphorus has nothing to do with phosphorescence, since, the glow of phosphorus is due to its chemical combination with atmospheric oxygen.

But in the case of solution of a luminescent substance treated in a similar manner the track of light appears cloudy, the colour of the track depending on the nature of the body.

In some cases of phosphorescent bodies, the glow continues for hours together while for other substances the body emits light only for small fraction of a second.

Becquerel invented an apparatus called phosphoroscope by which the duration of glow in the case of small times can be accurately determined. With this apparatus, it was found that all substances are phosphorescent to a greater or less extent.

**The Becquerel phosphoroscope.**—The instrument consists of a chamber blackened on the inside and provided with two windows at its two opposite faces. (Fig. IX-14). The substance to be examined is placed inside A. There are two discs D and E inside the chamber and fixed to the same axle XX. Each of the discs D and E are perforated with the same number of similar equi-spaced holes round its circumference. When the discs rotates, they shut and open the windows alternately. Since no two holes in the two discs are in the sameline of vision, light cannot pass straight through the windows B and C. It follows that when the substance in A is illuminated by light passing through B the vision at the window C is obstructed, and *vice versa*.

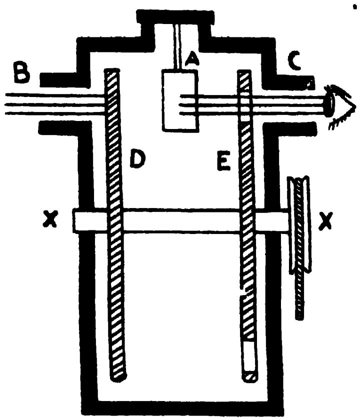



Fig. IX-14

By this means the duration of phosphorescene in the substance to be examined can be determined by adjusting the speed of rotation of the discs.